Mobile Communications

ECS 455

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Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

ECS 455 Chapter 1

Introduction & Review

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ECS 455 Chapter 1

Introduction & Review

1.1 Mobile Communications

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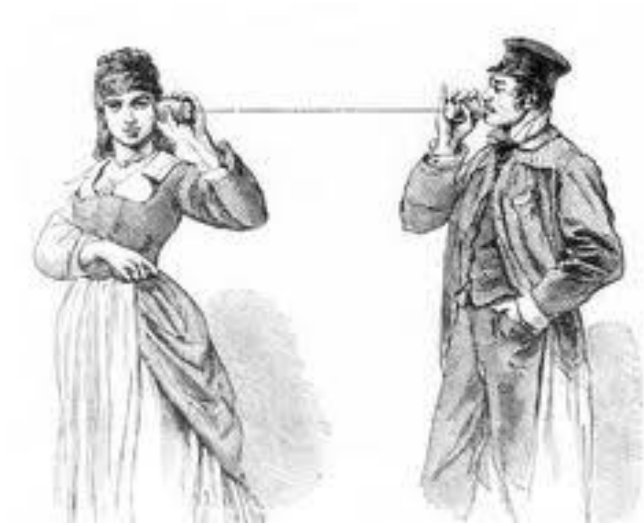
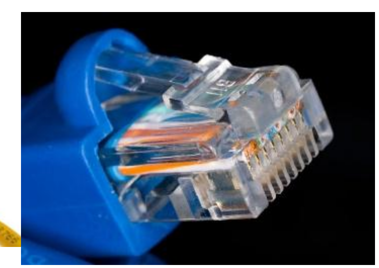
Wired Communication

- Cup-and-string communication





- POTS, Ethernet

Wireless communication

- Duncan Wilson's Cup Communicator

- Cellular Systems: 1G, 2G, 2.5G, 3G, **4G**
- Wireless LAN Systems: WiFi (802.11a/b/g/n/**ac**)

Overview of Mobile Communications

- Wireless/mobile communications is the **fastest growing** segment of the communications industry.
- Cellular systems have experienced **exponential growth** over the last decade.
- Cellular phones have become a critical business tool and part of everyday life in most developed countries, and are rapidly replacing wireline systems in many developing countries.

Mobile?

- The term "mobile" has historically been used to classify all radio terminal that could be moved during operation.
- More recently,
  - use "**mobile**" to describe a radio terminal that is attached to a high speed mobile platform
    - e.g., a cellular telephone in a fast moving vehicle
  - use "**portable**" to describes a radio terminal that can be hand-held and used by someone at **walking speed**
    - e.g., a walkie-talkie or cordless telephone inside a home.
    - 802.11?

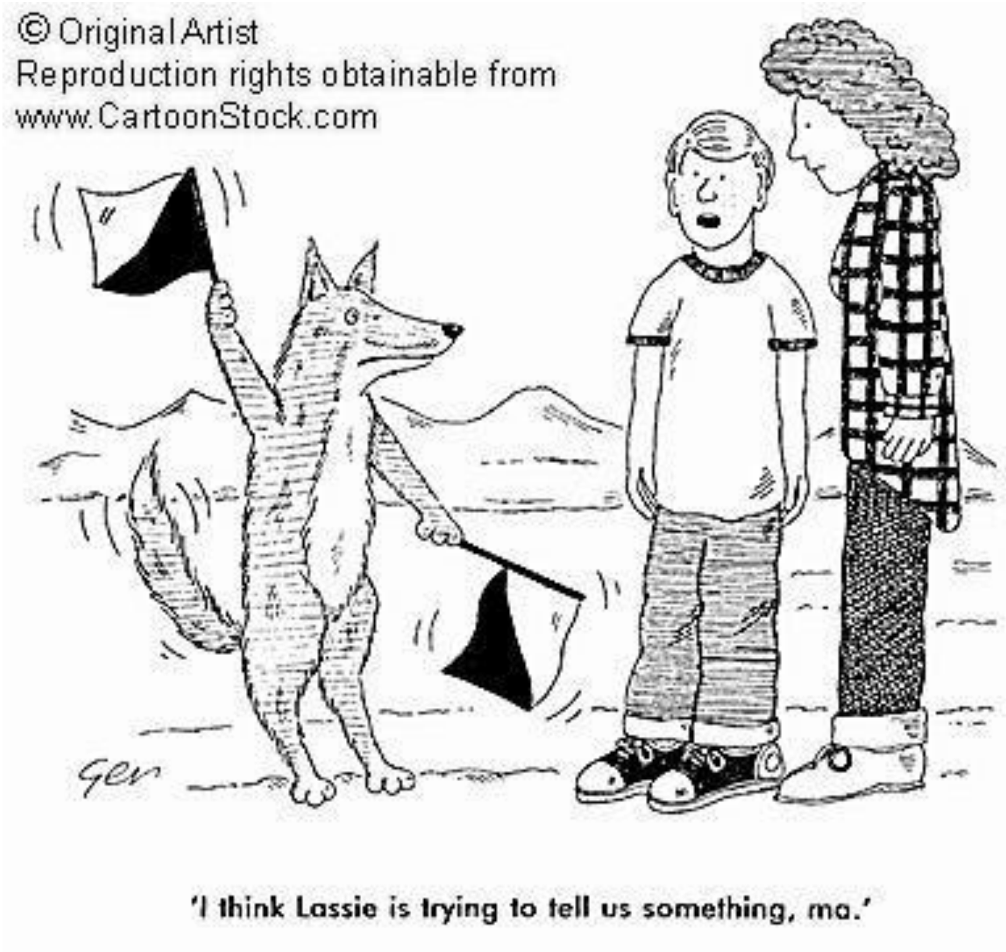
[Rappaport, 2002, p 9]

[Goldsmith, 2005, Section 1.1]

History (1)

- The first wireless networks were developed in the pre-industrial age.
- These systems transmitted information over **line-of-sight** distances (later extended by telescopes) using **smoke** signals, torch signaling, flashing mirrors, signal flares, or semaphore **flags**.

Semaphore



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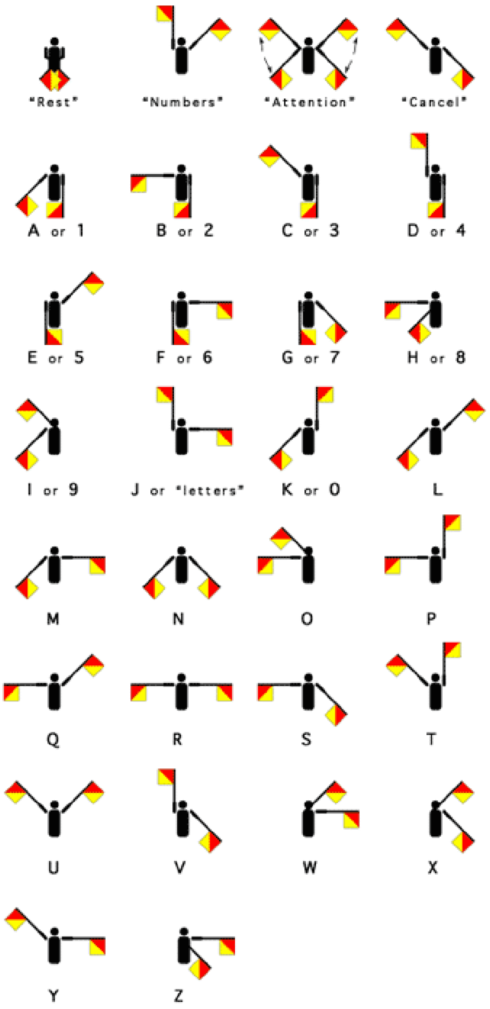
www.CartoonStock.com
'I think Lassie is trying to tell us something, ma.'



"Rest"
"Numbers"
"Attention"
"Cancel"
A or 1
B or 2
C or 3
D or 4
E or 5
F or 6
G or 7
H or 8
I or 9
J or "letters"
K or 0
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z

History: Radio

- Early communication networks were replaced first by the **telegraph network** (invented by Samuel **Morse** in 1838) and later by the telephone.
- In 1895, **Marconi** demonstrated the first radio transmission.
- Early radio systems transmitted **analog** signals.

- Today most radio systems transmit **digital** signals composed of binary bits.
- A digital radio can transmit a continuous bit stream or it can group the bits into packets.
- The latter type of radio is called a **packet radio** and is characterized by **bursty** transmissions

History: ALOHANET

- The first network based on packet radio, **ALOHANET**, was developed at the University of Hawaii in 1971.
- ALOHANET incorporated the first set of protocols for channel access and routing in packet radio systems, and many of the underlying principles in these protocols are still in use today.
- Lead to **Ethernet** and eventually wireless local area networks (**WLAN**).

History: Pre-Cellular (1)

- The **most successful** application of wireless networking has been the **cellular telephone system**.
- The roots of this system began in 1915, when wireless voice transmission between New York and San Francisco was first established.
- 1946: First public **mobile telephone** service was introduced in 25 cities across the United States.
- The equipment was expensive at $2,000
  - At that time was more than the price of a typical new car.

[Rappaport, 2002, p 4]

History: Pre-Cellular (2)

- These initial systems used a single **central transmitter** to cover an **entire** metropolitan **area**.
  - High-powered transmitter & Large tower
  - Inefficient!
  - FM push-to-talk
- 1976: (30 yrs after the introduction of the service in 1946),
  - the New York system (10M people) could only support 543 paying customers.
  - 3,700 on the waiting list
- The mobile units weighed about 10 kilograms and put out a steady 20-25 watts.
- The central transmitters that communicate with the mobile units broadcast 200 to 250 watts.

[Klemens, 2010, Chapter 3]

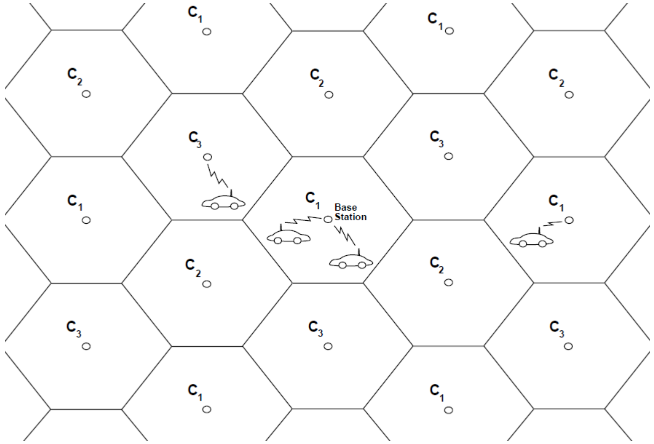
History: Pre-Cellular (3)

- The central station could reliably communicate with the mobile units up to a radius of approximately 25 miles (50 km).
- Beyond that, up to a radius of 60 to 100 miles, the signal was too weak for consistent service, but strong enough to interfere with any other mobile radio system.
- As a result, the central transmitters had to be at least 100 miles apart, leaving a 50 mile **blank space** between them.
- So a customer could use the sporadic and unreliable service only within the confines of one area.

[Klemens, 2010, p 46]

History: 1G Cellular (1)

- A solution to this capacity problem emerged during the 50's and 60's when researchers at AT&T **Bell Lab**oratories developed the **cellular concept**.
- 1968: AT&T proposed the concept to the FCC
- Cellular systems exploit the fact that the power of a transmitted signal falls off with distance.
- Thus, two users can operate on the same frequency at spatially-separate locations with minimal interference between them.
  - Frequency reuse



History: 1G Cellular (2)

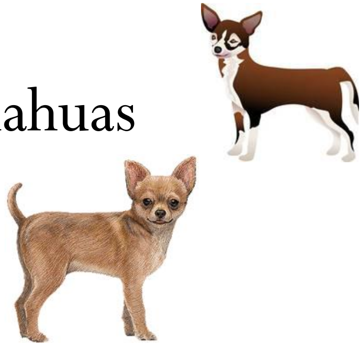
- **Japan** had the **world's first commercially available** cellular phone system.
  - Nippon Telegraph and Telephone (NTT) created a cellular test system for Tokyo in 1975, with the result coming to market in 1979.
- The first trial in America of a complete, working cellular system was held in **Chicago** in the late 1970's.
- 1983: Advanced Mobile Phone System (**AMPS**)
  - First US cellular telephone system
  - Deployed in 1983 by Ameritech in Chacago, IL.
  - Worked well. (FM, FDMA)
  - May even have worked too well.
    - Its satisfactory performance lowered the demand for a better system, allowing Europe to take the lead by creating a digital cellular system first.

Old Cell Phone

Motorola's DynaTAC

First **commercially available** cell phone in 1983

- Weighed about 2 lbs (1 Kg)
- 10 inches high, making it larger than some Chihuahuas
- Battery life: 30 minutes of talk time
- $4,000



History: 2G Cellular

- The **first-generation (1G)** systems introduced in the 1980s were characterized by **analog speech transmission**.
- The second generation (**2G**) of cellular systems, first deployed in the early 1990's, were based on **digital** communications.
- The shift from analog to digital was driven by its higher capacity and the improved cost, speed, and power efficiency of digital hardware.
- 1991: US Digital Cellular (**USDC** – **IS-54** > IS-136)
  - Three times capacity of AMPS because digital modulation, speech coding, and TDMA
- While second generation cellular systems initially provided mainly **voice** services, these systems gradually evolved to support **data** services such as email, Internet access, and short messaging.

Two important 2G systems

- **GSM** supports SMSs and user data at rates only up to **9.6 kb/s**.
  - Security features including (for example) the encryption of data and signaling messages on the path between the mobile phone and the BS.
  - Subscriber identity module (SIM)
    - A smart card
    - Contain the subscriber's personal details
    - Can be moved from one handset to another.

- **IS-95B** (**cdmaOne**) provides data rates in the range of **64 to 115 kb/s** in increments of 8 kb/s over a 1.25 MHz channel.
  - Each cell uses a carrier with a bandwidth of 1.25MHz, which is divided into 64 data and signalling channels by the use of orthogonal CDMA codes.

History: 2G Standard Proliferation

- Unfortunately, the **great market potential** for cellular phones led to a proliferation of (incompatible) second generation cellular standards.
- As a result of the **standard proliferation**, many cellular phones today are **multi-mode**.

Major Mobile Radio Standards in North America

Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modulation	Channel Bandwidth
AMPS	Cellular	1983	FDMA	824-894 MHz	FM	30 kHz
NAMPS	Cellular	1992	FDMA	824-894 MHz	FM	10 kHz
USDC	Cellular	1991	TDMA	824-894 MHz	$\pi/4$-DQPSK	30 kHz
CDPD	Cellular	1993	FH/ Packet	824-894 MHz	GMSK	30 kHz
IS-95	Cellular/ PCS	1993	CDMA	824-894 MHz 1.8-2.0 GHz	QPSK/ BPSK	1.25 MHz
GSC	Paging	1970s	Simplex	Several	FSK	12.5 kHz
POCSAG	Paging	1970s	Simplex	Several	FSK	12.5 kHz
FLEX	Paging	1993	Simplex	Several	4-FSK	15 kHz
DCS-1900 (GSM)	PCS	1994	TDMA	1.85-1.99 GHz	GMSK	200 kHz
PACS	Cordless/ PCS	1994	TDMA/ FDMA	1.85-1.99 GHz	$\pi/4$-DQPSK	300 kHz
MIRS	SMR/PCS	1994	TDMA	Several	16-QAM	25 kHz
iDen	SMR/PCS	1995	TDMA	Several	16-QAM	25 kHz

[Rappaport, 2002, Table 1.1]

Major Mobile Radio Standards in Europe

Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modulation	Channel Bandwidth
JTACS	Cellular	1988	FDMA	860-925 MHz	FM	25 kHz
PDC	Cellular	1993	TDMA	810-1501 MHz	π/4-DQPSK	25 kHz
NTT	Cellular	1979	FDMA	400/800 MHz	FM	25 kHz
NTACS	Cellular	1993	FDMA	843-925 MHz	FM	12.5 kHz
NTT	Paging	1979	FDMA	280 MHz	FSK	12.5 kHz
NEC	Paging	1979	FDMA	Several	FSK	10 kHz
PHS	Cordless	1993	TDMA	1895-1907 MHz	π/4-DQPSK	300 kHz

[Rappaport, 2002, Table 1.2]

Major Mobile Radio Standards in Japan

Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modula-tion	Channel Bandwidth
ETACS	Cellular	1985	FDMA	900 MHz	FM	25 kHz
NMT-450	Cellular	1981	FDMA	450-470 MHz	FM	25 kHz
NMT-900	Cellular	1986	FDMA	890-960 MHz	FM	12.5 kHz
GSM	Cellular /PCS	1990	TDMA	890-960 MHz	GMSK	200 kHz
C-450	Cellular	1985	FDMA	450-465 MHz	FM	20 kHz/ 10 kHz
ERMES	Paging	1993	FDMA	Several	4-FSK	25 kHz
CT2	Cordless	1989	FDMA	864-868 MHz	GFSK	100 kHz
DECT	Cordless	1993	TDMA	1880-1900 MHz	GFSK	1.728 MHz
DCS-1800	Cordless /PCS	1993	TDMA	1710-1880 MHz	GMSK	200 kHz

[Rappaport, 2002, Table 1.3]

History (Thailand)

- 1G
  - 1986 (2529): NMT470 (TOT)
    - Nordic Mobile Telephone System @ 470MHz
  - AMPS (Advanced Mobile Phone System)
    - 1990 (2533): Cellular 900 (AIS)
    - Worldphone 800 (TAC)
- 2G: GSM (Global System for Mobile Communication)
  - 2537: GSM Advance @ 900 Mhz (AIS)
  - Worldphone 1800 (TAC)

NMT450

[http://3g.siamphone.com/articles/2009/3g/page.htm]

2.5G: GSM Enhancement

- Want to deliver ***data*** as well as voice.
- **General Packet Radio Service (GPRS)**
- **Enhanced Data Rates for GSM Evolution (EDGE)**

2.5G: GPRS

- **General Packet Radio Service**
- The first commercial launches for GPRS took place in 2001.
- Provide connectivity to IP networks (Internet).
- Construction of a **packet switched** core network, to run alongside the **circuit switched** network that was originally built for GSM.
  - "always on" connection that remains active as long as the phone is within range of the service.
- A single time slot may be shared by multiple users for transferring packet mode data
- Each slot can handle up to **20 kb/s**. Each user may be allocated up to 8 slots
  - Data rates up to about **160 kb/s** per user are possible.
  - A good approximation for throughput in "average" conditions is 10 Kbps per time slot. [Korhonen, 2003]

2.75?G: EDGE

- **Enhanced Data Rates for GSM Evolution**
  - Originally this acronym stood for Enhanced Data rates for GSM Evolution, but now it translates into **Enhanced Data rates for Global Evolution**, as the EDGE idea can also be used in systems other than GSM [Korhonen, 2003]
- Support IP-based services in GSM at rates up to **384 kb/s**
- Only requires a **software upgrade** to base stations
  - if the RF amplifiers can handle the non-constant envelope modulation with EDGE's relatively high peak-to-average power ratio (PAPR).
- EDGE is popular in North America, where the allocation of carrier frequencies has made it hard for GSM operators to upgrade to UMTS.

Motivation

Voice/SMS

~9.6Kbps

Mobile Narrow Band Internet

14.4~64Kbps

Low-QoS Mobile Multimedia Services

64~144Kbps

High-quality, Smooth and Low-delay Video, Voice, and Music Services

20~300Kbps

Mobile Broadband Internet Surfing

64~300Kbps

Abundant and High-quality New Mobile Services

300K~5Mbps

3G

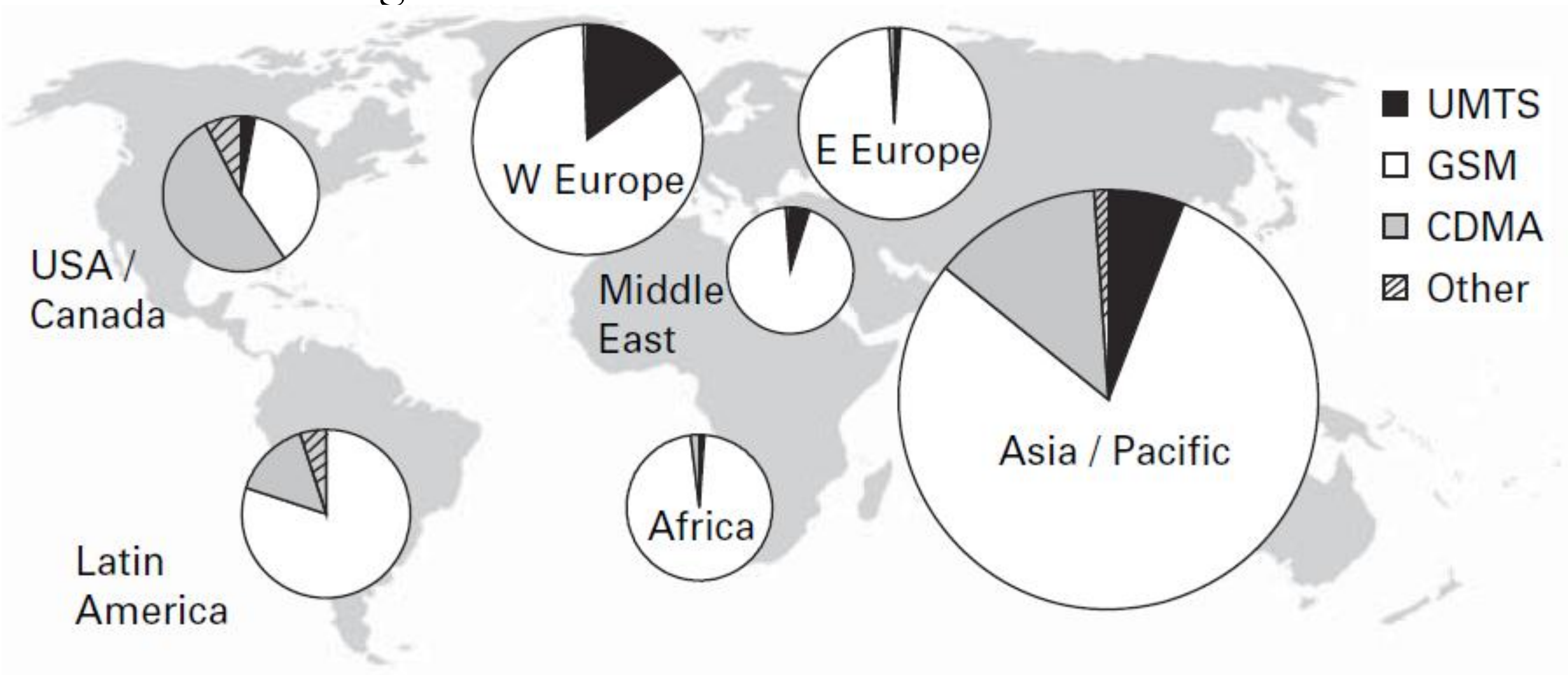
- International Mobile Telecommunications-2000 (**IMT-2000**)
  - A subgroup of the International Telecommunication Union (**ITU**)
  - Published a set of performance requirements of 3G (for both packet-switched and circuit-switched data):
    - A minimum data rate of 144 Kbps in the vehicular environment
    - A minimum data rate of 384 Kbps in the pedestrian environment
    - A minimum data rate of 2 Mbps in the fixed indoor and picocell environment
- There are several wireless standards and systems that qualify as third generation (3G) systems
  - UMTS
  - CDMA2000

UMTS

- **Universal Mobile Telecommunications System** (**UMTS**)
- The research activity on UMTS started in **Europe** at the beginning of the 1990s.
  - Even before the earliest 2G systems arrived on the market
- Designed to support wideband services with data rates up to **2Mbit/s**.
- Developed **from GSM**
  - Keep the core network more-or-less intact
  - Change the air interface to use **CDMA**
- Compatibility between UMTS and GSM:
  - Most UMTS mobiles also implement GSM, and the network can hand them over from a UMTS base station to a GSM one if they reach the edge of the UMTS coverage area.
  - However, network operators cannot implement the two systems in the same frequency band, so they are not fully compatible with each other.

Market Share

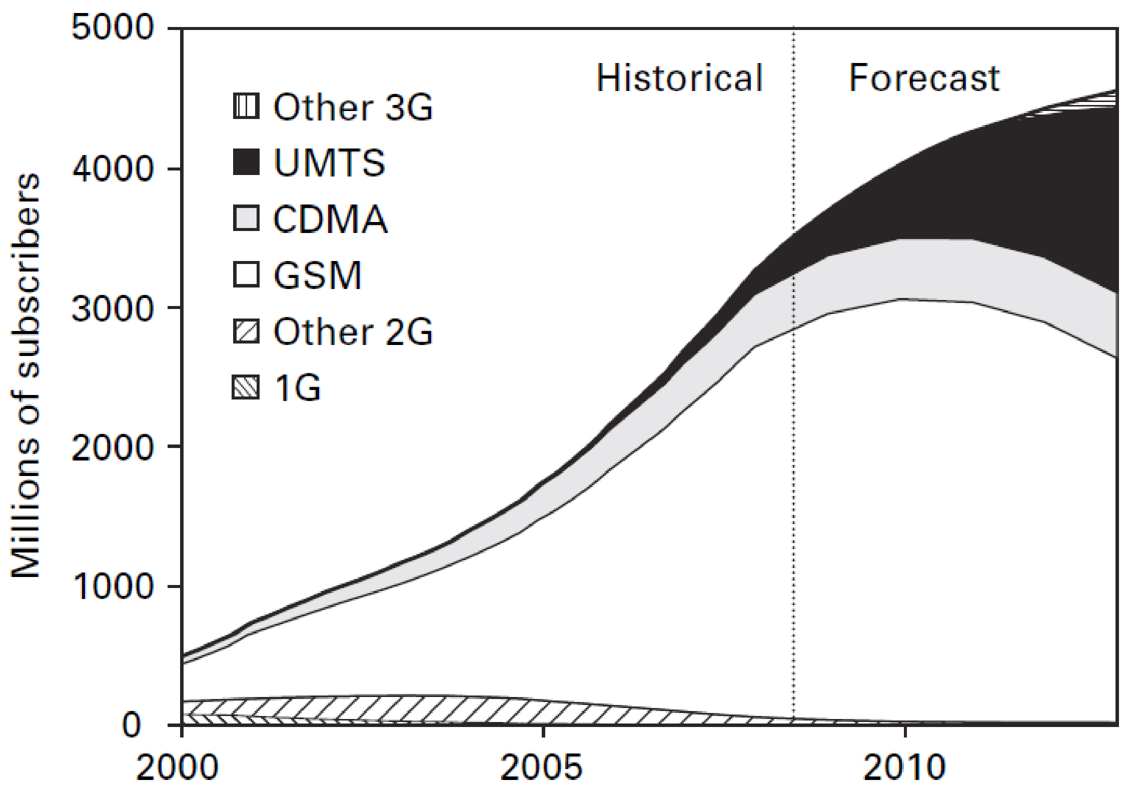
- Numbers of subscribers to different mobile communication technologies in 2008.



[Cox, 2008, Fig 1.15]

Growth

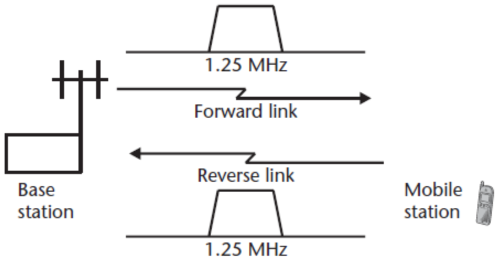
- Growth in the use of different mobile telecommunication technologies, with historical data from 2000 to 2008, and forecasts from 2008 to 2013.



[Cox, 2008, Fig 1.16]

cdma2000

- Another 3G mobile technology standard
- Multicarrier, direct-sequence CDMA FDD system.
- Backward-compatible with its previous 2G iteration IS-95 (cdmaOne).
- **CDMA2000 1X** (**IS-2000**)
  - also known as 1x and 1xRTT
  - 1x = Spreading Rate 1 = use the same chip rate of IS-95 (i.e., 1.2288 Mcps).
    - Same RF bandwidth as IS-95: a duplex pair of 1.25 MHz radio channels.
  - Core CDMA2000 wireless air interface standard.
  - Almost doubles the capacity of IS-95 by adding 64 more traffic channels to the forward link, orthogonal to (in quadrature with) the original set of 64.



Evolution of UMTS Specifications

Release	Functional freeze	Main UMTS feature of release
Rel-99	March 2000	Basic 3.84 Mcps W-CDMA (FDD & TDD)
Rel-4	March 2001	1.28 Mcps TDD (aka TD-SCDMA)
Rel-5	June 2002	HSDPA
Rel-6	March 2005	HSUPA (E-DCH)
Rel-7	December 2007	HSPA+ (64QAM downlink, MIMO, 16QAM uplink) LTE and SAE feasibility study

Also dubbed 3.5G, 3G+ or turbo 3G

HSPA

3.5G?

- **High Speed Packet Access (HSPA)** is a collection of two mobile telephony protocols
  - High Speed **Downlink** Packet Access (HSDPA) and
  - High Speed **Uplink** Packet Access (HSUPA)
- Extend and improve the performance of existing WCDMA/UMTS protocols.
- Current HSDPA deployments support down-link speeds of 1.8, 3.6, **7.2** and 14.0 Megabit/s.
- Many HSPA rollouts can be achieved by a **software upgrade** to existing 3G networks, giving HSPA a head start over WiMAX, which requires dedicated network infrastructure.
- There is also a further standard, **Evolved HSPA (HSPA+)**. 3.9G?
  - HSPA+ provides speeds of up to **42 Mbit/s** downlink and 84 Mbit/s with Release 9 of the 3GPP standards.

3G in Thailand: HSPA, HSPA+

- TOT: 2.1 GHz
- Truemove: 850 MHz
- Dtac: 850 MHz
- AIS: 900 Mhz
- Be careful!

Iphone 4

- GSM model:
  - 2G: Quadband GSM/EDGE (850, 900, 1800, 1900 MHz)
  - 3G: UMTS/HSDPA (7.2 Mbps)/HSUPA (850, 900, 1900, 2100 MHz)
- CDMA model: CDMA EV-DO Rev. A (800, 1900 MHz)
- Wi-Fi 802.11b/g/n
  - WLAN (Wireless LAN)
  - 802.11n 2.4GHz only
- Bluetooth v2.1+ EDR
- A-GPS navigation

Samsung Galaxy SII

- The version released on Sprint will be called the Samsung Within, on AT&T, the Attain, and on Verizon, the Function.
- Two models for 3G (HSPA+)
  - 850/2100 MHz
  - 900/2100 MHz
- Wi-Fi 802.11b/g/n
- Bluetooth 3.0

“4G” in the US

Sprint
Mobile WiMax
Sprint
Seattle
Spokane
Portland
Billings
Omaha
Reno
Kansas City
Wichita
Indianapolis
Louisville
New York
Virginia Beac
Oklahoma City
Little Rock
Phoenix
Tucson
San Antoni

verizonwireless
LTE
verizon

T-Mobile
HSPA+
T-Mobile

Comparison

	Sprint	verizon	T · ·Mobile·
"4G" Technology	WiMax	LTE	HSPA+
Speed	Downstream average speeds of 3-6 Mbps,10 Mbps peak. "10x faster than 3G."	5-12 Mbps downstream and 2-5 Mbps upstream	Peak downstream of 21Mbps, peak upstream of 5.7Mbps. "Up to 3x faster than 3G."
Coverage	Nationwide coverage of many major cities and markets.	38 markets and 60 major airports on December 5th, 2010. Full nation-wide cover-age by 2013.	Nationwide coverage of many major cities and markets.
OS Compatibility	All OS' (via Mobile HotSpot)	Windows-only (at launch, Mac OS X support to come)	Windows and Mac
Devices	USB Modems and Mobile Hotspot	USB Modems	USB Modems and Netbook
Monthly Cost	$60 for unlimited 4G data and 5GB of 3G data.	$50 for 5GB, $80 for 10GB, $10/GB overage fee.	$25 for 200MB, $40 for 5GB and no ovarage fees.

[lifehacker.com]

LTE: Around the World

[Wireless Week Magazine, February 2011]

ECS 455 Chapter 1

Introduction & Review

1.2 Fourier Transform and Communication System

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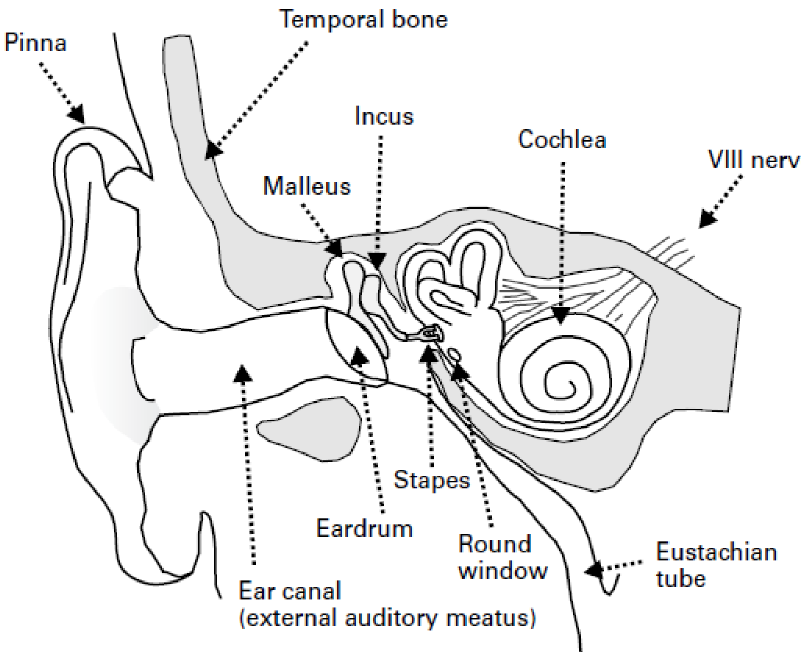
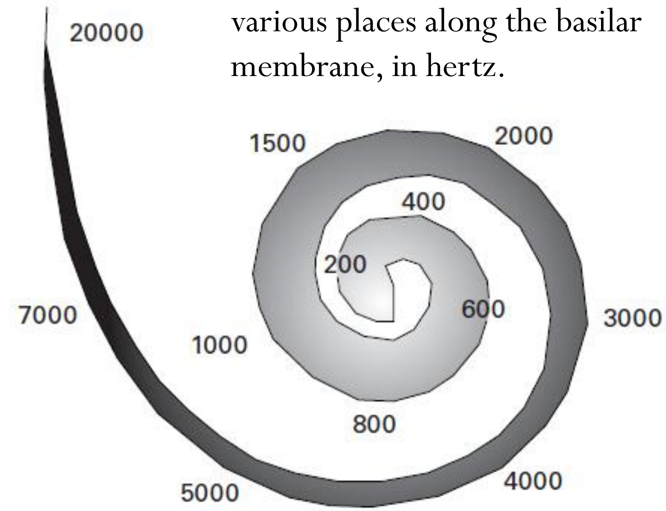
The cochlea has sometimes been described as a **biological Fourier analyzer**.

Fourier Transform in Auditory System



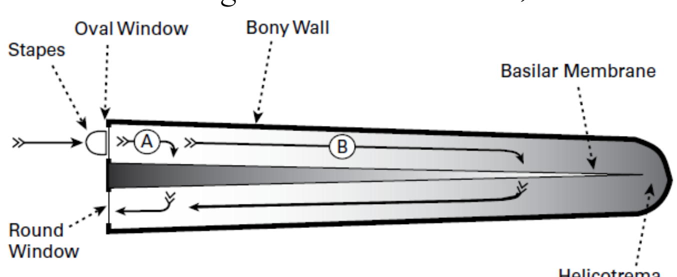
[Schnupp, Nelken, and King, 2010, Fig 2.1]

Approximate best frequencies of various places along the basilar membrane, in hertz.

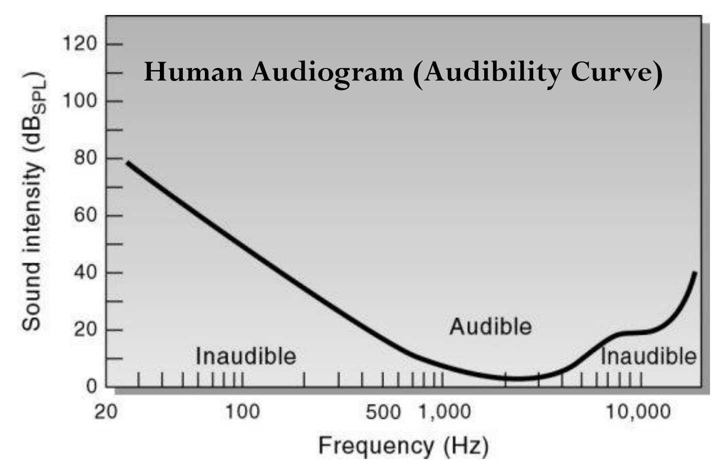


[Schnupp, Nelken, and King, 2010, Fig 2.2]

Schematic showing the cochlea unrolled, in cross-section.



[Schnupp, Nelken, and King, 2010, Fig 2.2]



[http://psyc254.uconn.edu/Lecture18/]

Notes #1

- Fourier Transform
- Modulation
- DSB-SC and QAM

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ECS 455: Mobile Communications
Fourier Transform and Communication Systems

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January 9, 2012

Communication systems are usually viewed and analyzed in frequency domain. This note reviews some basic properties of Fourier transform and introduce basic communication systems.

Contents

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Contents

1 Introduction to communication systems

1.1. Shannon's insight [5]:

> The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.

Definition 1.2. Figure 1 [5] shows a commonly used model for a (single-link or point-to-point) communication system. All information transmission systems involve three major subsystems–a transmitter, the channel, and a receiver.

(a) **Information** source: produce a **message**

- Messages may be categorized as **analog** (continuous) or **digital** (discrete).

(b) **Transmitter**: operate on the message to create a **signal** which can be sent through a channel

(c) **Channel**: the medium over which the signal, carrying the information that composes the message, is sent

- All channels have one thing in common: the signal undergoes **degradation** from transmitter to receiver.
  - Although this degradation may occur at any point of the communication system block diagram, it is customarily associated with the channel alone.
  - This degradation often results from noise and other undesired signals or interference but also may include other distortion effects as well, such as fading signal levels, multiple transmission paths, and filtering.

(d) **Receiver**: transform the signal back into the message intended for delivery

(e) **Destination**: a person or a machine, for whom or which the message is intended

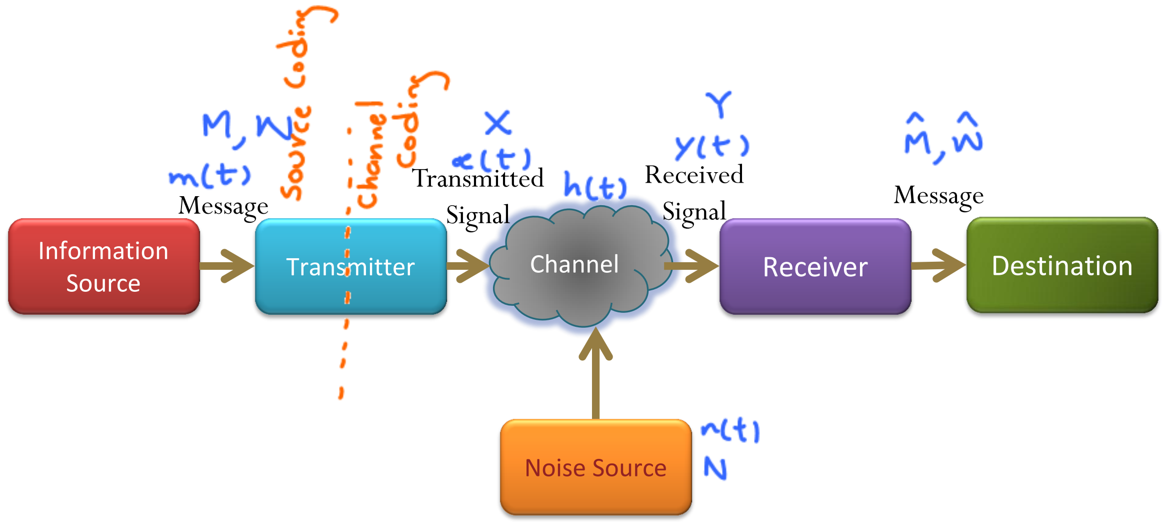


Figure 1: Schematic diagram of a general communication system

2 Frequency-Domain Analysis

Electrical engineers live in the two worlds, so to speak, of time and frequency. Frequency-domain analysis is an extremely valuable tool to the communications engineer, more so perhaps than to other systems analysts. Since the communications engineer is concerned primarily with signal bandwidths and signal locations in the frequency domain, rather than with transient analysis, the essentially steady-state approach of the (complex exponential) **Fourier series** and **transforms** is used rather than the Laplace transform.

2.1 Math background

2.1. ***Euler's formula***: $e^{jx} = \cos x + j \sin x$.

$$\cos(A) = \text{Re}\left\{e^{jA}\right\} = \frac{1}{2}\left(e^{jA} + e^{-jA}\right)$$

$$\sin(A) = \text{Im}\left\{e^{jA}\right\} = \frac{1}{2j}\left(e^{jA} - e^{-jA}\right).$$

2.2. We can use $\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$ and $\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$ to derive many trigonometric identities.

Example 2.3. $\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$

2.4. Similar technique gives

(a) $\cos(-x) = \cos(x)$,

(b) $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$,

(c) $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$, and

(d) the **product-to-sum formula**

$$\cos(x)\cos(y) = \frac{1}{2}\left(\cos(x+y) + \cos(x-y)\right). \tag{1}$$

2.2 Continuous-Time Fourier Transform

Definition 2.5. The (direct) **Fourier transform** of a signal $g(t)$ is defined by

$$G(f) = \int_{-\infty}^{+\infty} g(t)e^{-j2\pi ft}dt \tag{2}$$

This provides the frequency-domain description of $g(t)$. Conversion back to the time domain is achieved via the **inverse (Fourier) transform**:

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi ft}df \tag{3}$$

- We may combine (2) and (3) into one compact formula:

$$\boxed{\int_{-\infty}^{\infty} G(f)\, e^{j2\pi ft}df = g(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi ft}dt.} \tag{4}$$

- We may simply write $G = \mathcal{F}\{g\}$ and $g = \mathcal{F}^{-1}\{G\}$.
- Note that $G(0) = \int g(t)dt$ and $g(0) = \int G(f)df$.

2.6. In some references[1], the (direct) Fourier transform of a signal $g(t)$ is defined by

$$G_2(\omega) = \int_{-\infty}^{+\infty} g(t)e^{-j\omega t}dt \tag{5}$$

[1] MATLAB uses this definition.

In which case, we have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} G_2(\omega)\, e^{j\omega t} d\omega = g(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G_2(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t} dt \tag{6}$$

- In MATLAB, these calculations are carried out via the commands `fourier` and `ifourier`.
- Note that $\hat{G}(0) = \int g(t)dt$ and $g(0) = \frac{1}{2\pi}\int G(\omega)d\omega$.
- The relationship between $G(f)$ in (2) and $G_2(\omega)$ in (5) is given by

$$G(f) = G_2(\omega)\big|_{\omega=2\pi f} \tag{7}$$

$$G_2(\omega) = G(f)\big|_{f=\frac{\omega}{2\pi}} \tag{8}$$

2.7. Q: The relationship between $G(f)$ in (2) and $G_2(\omega)$ in (5) is given by (7) and (8) which do not involve a factor of 2π in the front. Why then does the factor of $\frac{1}{2\pi}$ shows up in (6)?

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t} df = \int_{-\infty}^{\infty} G\left(\frac{\omega}{2\pi}\right) e^{j2\pi \frac{\omega}{2\pi} t} d\frac{\omega}{2\pi}$$

$\uparrow$ $\omega = 2\pi f$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} G_2(\omega)\, e^{j\omega t} d\omega$$

A: From changing $df \to d\omega$, we need a factor of $\frac{1}{2\pi}$.

Example 2.8. Rectangular and Sinc: $G(f)$

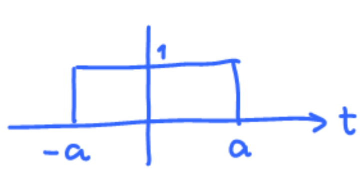
$$g(t) = 1[|t| \le a] \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{\sin(2\pi f a)}{\pi f} = \frac{2\sin(a\omega)}{\omega} = 2a\ \operatorname{sinc}(a\omega) \tag{9}$$

$1_{[-a,a]}(t)$

Indicator function

$$1_A(t) = \begin{cases} 1, & t \in A \\ 0, & t \notin A \end{cases}$$

$\uparrow$ set, interval



$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t} dt = \int_{-\infty}^{\infty} 1_{[-a,a]}(t)\, e^{-j2\pi f t} dt$$

$$= \int_{-a}^{a} e^{-j2\pi f t} dt = \frac{1}{-j2\pi f} e^{-j2\pi f t}\Big|_{-a}^{a}$$

$$= \frac{e^{-j2\pi f a} - e^{-j2\pi f(-a)}}{-j2\pi f} = \frac{e^{j2\pi f a} - e^{-j2\pi f a}}{j2\pi f}$$

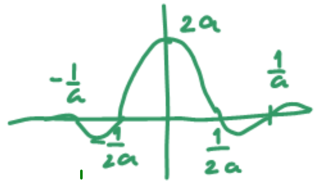
$$= \frac{2a\, \cancel{2j} \sin(2\pi f a)}{2a\, \cancel{j2}\pi f} = 2a\ \operatorname{sinc}(2\pi f a)$$

$\uparrow$ $\operatorname{sinc}(x) = \frac{\sin x}{x}$

$\sin x = 0$

$x = n\pi$

$2\pi f a = n\pi \Rightarrow f = \frac{1}{2a} \times n$



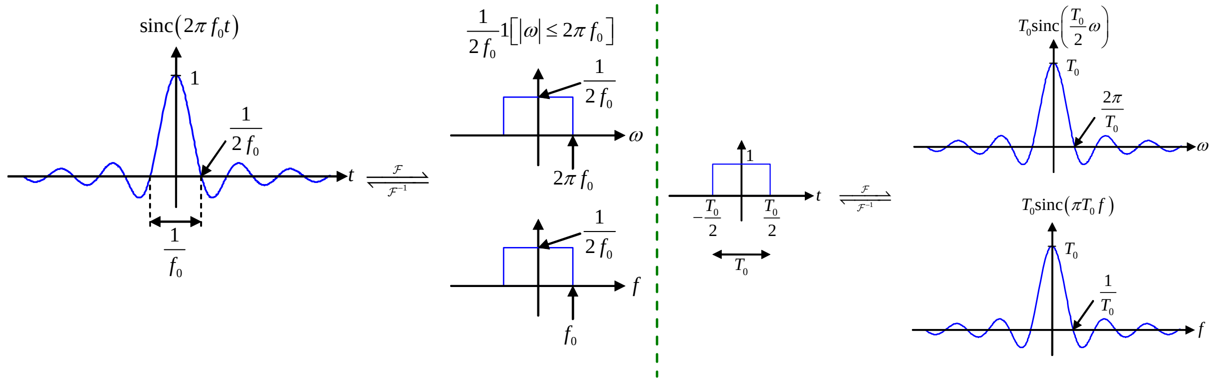


Figure 2: Fourier transform of sinc and rectangular functions

- By setting $a = T_0/2$, we have

$$1\left[|t| \leq \frac{T_0}{2}\right] \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} T_0 \operatorname{sinc}(\pi T_0 f). \tag{10}$$

- In [2, p 78], the function $1[|t| \leq 0.5]$ is defined as the **unit gate** function $\operatorname{rect}(x)$.

Definition 2.9. The function $\operatorname{sinc}(x) \equiv (\sin x)/x$ is plotted in Figure 3.

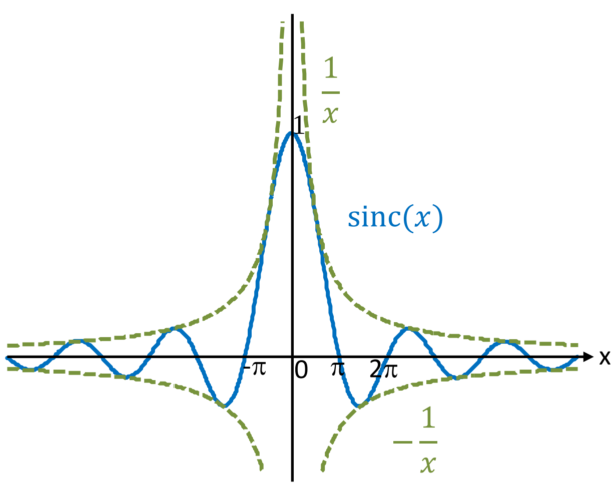


Figure 3: Sinc function

- This function plays an important role in signal processing. It is also known as the filtering or interpolating function.

- Using L'Hôpital's rule, we find $\lim_{x \to 0} \operatorname{sinc}(x) = 1$.
- $\operatorname{sinc}(x)$ is the product of an oscillating signal $\sin(x)$ (of period 2π) and a monotonically decreasing function $1/x$. Therefore, $\operatorname{sinc}(x)$ exhibits sinusoidal oscillations of period 2π, with amplitude decreasing continuously as $1/x$.
- In `MATLAB` and in [7, eq. 2.64], $\operatorname{sinc}(x)$ is defined as $(\sin(\pi x))/\pi x$. In which case, it is an even damped oscillatory function with zero crossings at integer values of its argument.

Definition 2.10. The (Dirac) **delta function** or (unit) impulse function is denoted by $\delta(t)$. It is usually depicted as a vertical arrow at the origin. Note that $\delta(t)$ is not a true function; it is undefined at $t = 0$. We define $\delta(t)$ as a generalized function which satisfies the **sampling property** (or **sifting property**)

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0) \tag{11}$$

for any function $\phi(t)$ which is continuous at $t = 0$. From this definition, It follows that

$$(\delta * \phi)(t) = (\phi * \delta)(t) = \int_{-\infty}^{\infty} \phi(\tau)\delta(t - \tau)d\tau = \phi(t) \tag{12}$$

where we assume that ϕ is continuous at t.

- Intuitively we may visualize $\delta(t)$ as an infinitely tall, infinitely narrow rectangular pulse of unit area: $\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} 1\left[|t| \le \frac{\varepsilon}{2}\right]$.

2.11. Properties of $\delta(t)$:

- $\delta(t) = 0$ when $t \neq 0$.
  $\delta(t - T) = 0$ for $t \neq T$.
- $\int_A \delta(t)dt = 1_A(0)$.
  (a) $\int \delta(t)dt = 1$.
  (b) $\int_{\{0\}} \delta(t)dt = 1$.
  (c) $\int_{-\infty}^{x} \delta(t)dt = 1_{[0,\infty)}(x)$. Hence, we may think of $\delta(t)$ as the "derivative" of the unit step function $U(t) = 1_{[0,\infty)}(x)$.

- $\int \phi(t)\delta(t)dt = \phi(0)$ for ϕ continuous at 0.
- $\int \phi(t)\delta(t-T)dt = \phi(T)$ for ϕ continuous at T. In fact, for any $\varepsilon > 0$,

$$\int_{T-\varepsilon}^{T+\varepsilon} \phi(t)\delta(t-T)dt = \phi(T).$$

- $\delta(at) = \frac{1}{|a|}\delta(t)$. In particular,

$$\delta(\omega) = \frac{1}{2\pi}\delta(f) \tag{13}$$

$\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$

and

$$\delta(\omega - \omega_0) = \delta(2\pi f - 2\pi f_0) = \frac{1}{2\pi}\delta(f - f_0), \tag{14}$$

where $\omega = 2\pi f$ and $\omega_0 = 2\pi f_0$.

Example 2.12. $\delta(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} 1$.

$\mathcal{F}$: $\int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt = e^{-j2\pi f 0} = 1$

Example 2.13. $e^{j2\pi f_0 t} \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \delta(f - f_0)$.

$\mathcal{F}^{-1}$: $\int_{-\infty}^{\infty} \delta(f-f_0) e^{j2\pi f t} df = e^{j2\pi f t}\Big|_{f=f_0} = e^{j2\pi f_0 t}$

Example 2.14. $e^{j\omega_0 t} \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} 2\pi\delta(\omega - \omega_0)$.

$e^{j2\pi f_0 t} \rightarrow \delta(f - f_0)$ (14)

$= e^{j\omega_0 t}$

Example 2.15. $\cos(2\pi f_0 t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$.

$= \frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$

2.16. Conjugate symmetry[2]: If $x(t)$ is **real**-valued, then $X(-f) = (X(f))^*$

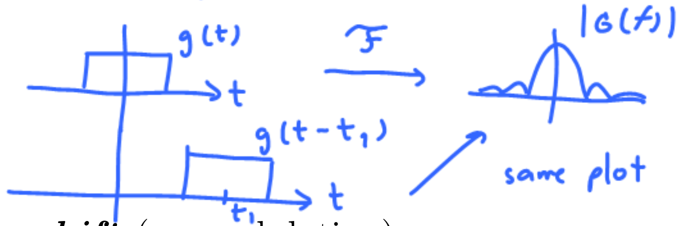
Observe that if we know $X(f)$ for all f positive, we also know $X(f)$ for all f negative. Interpretation: Only half of the spectrum contains all of the information. Positive-frequency part of the spectrum contains all the necessary information. The negative-frequency half of the spectrum can be determined by simply complex conjugating the positive-frequency half of the spectrum.

2.17. ***Shifting*** properties

- ***Time-shift***:

$$g(t-t_1) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} e^{-j2\pi f t_1} G(f)$$

  - Note that $|e^{-j2\pi f t_1}| = 1$. So, the spectrum of $g(t-t_1)$ looks exactly the same as the spectrum of $g(t)$ (unless you also look at their phases).



- ***Frequency-shift*** (or modulation):

$$e^{j2\pi f_1 t} g(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G(f-f_1)$$

[2]Hermitian symmetry in [4, p 17].

2.18. Let $g(t)$, $g_1(t)$, and $g_2(t)$ denote signals with $G(f)$, $G_1(f)$, and $G_2(f)$ denoting their respective Fourier transforms.

(a) ***Superposition theorem*** (linearity): followed directly from linearity of integration.

$$a_1 g_1(t) + a_2 g_2(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} a_1 G_1(f) + a_2 G_2(f).$$

(b) ***Scale-change*** **theorem** (scaling property [2, p 88]):

for $a > 1$ ⇒ compression in time

$$g(at) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{|a|} G\left(\frac{f}{a}\right).$$

← expanded in freq. domain

- The function $g(at)$ represents the function $g(t)$ *compress*ed in time by a factor a (when $|a| > l$). Similarly, the function $G(f/a)$ represents the function $G(f)$ *expand*ed in frequency by the same factor a.
- The scaling property says that if we "squeeze" a function in t, its Fourier transform "stretches out" in f. It is not possible to arbitrarily concentrate both a function and its Fourier transform.
- Generally speaking, the more concentrated $g(t)$ is, the more spread out its Fourier transform $G(f)$ must be.
- This trade-off can be formalized in the form of an *uncertainty principle*. See also 2.28 and 2.29.
- Intuitively, we understand that compression in time by a factor a means that the signal is varying more rapidly by the same factor. To synthesize such a signal, the frequencies of its sinusoidal components must be increased by the factor a, implying that its frequency spectrum is expanded by the factor a. Similarly, a signal expanded in time varies more slowly; hence, the frequencies of its components are lowered, implying that its frequency spectrum is compressed.

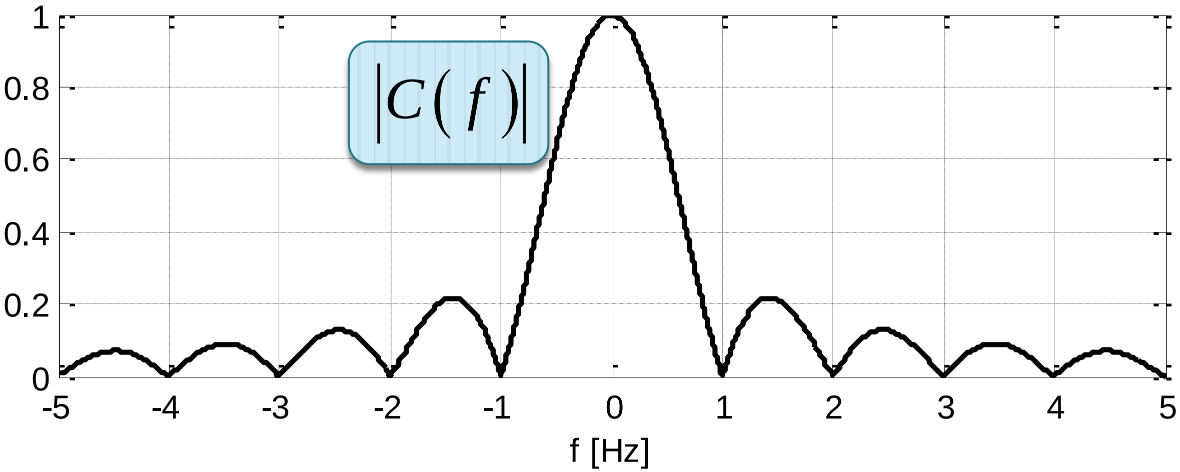
(c) ***Duality theorem*** (Symmetry Property [2, p 86]):

$g(t) \xrightarrow{\mathcal{F}} G(f)$

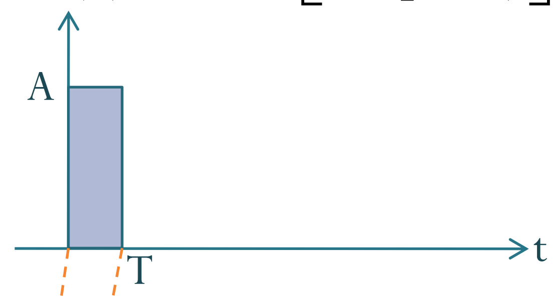
$$G(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} g(-f).$$

Spectrum of Digital Data (1/4)

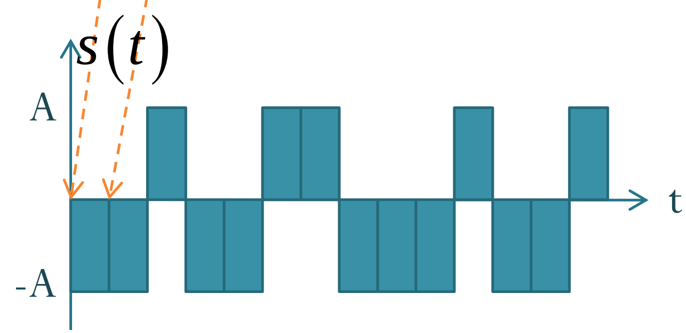
$(A=1, T=1)$



$$c(t) = A \times 1\left[t \in [0,T)\right]$$



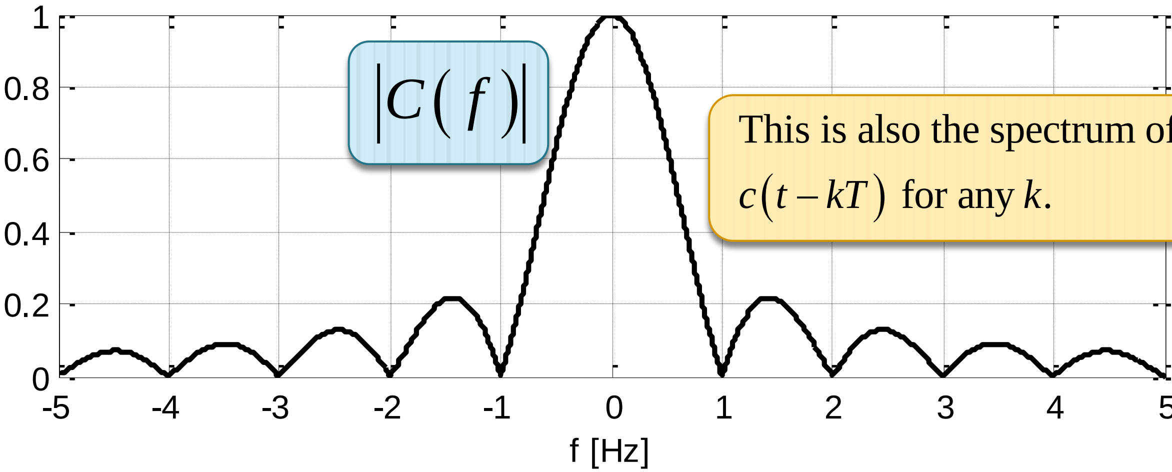
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



Can you sketch the spectrum of $s(t)$?

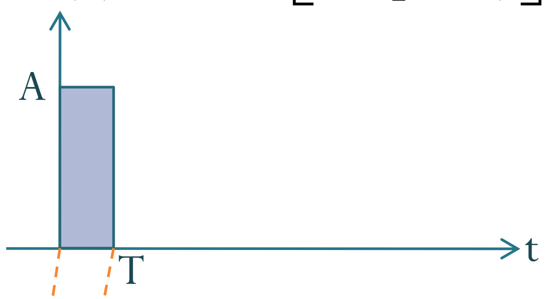
Spectrum of Digital Data (2/4)

$(A=1, T=1)$



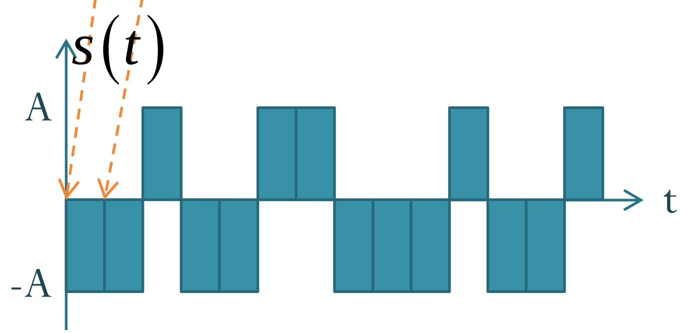
This is also the spectrum of $c(t-kT)$ for any k.

$$c(t) = A \times 1[t \in [0,T)]$$



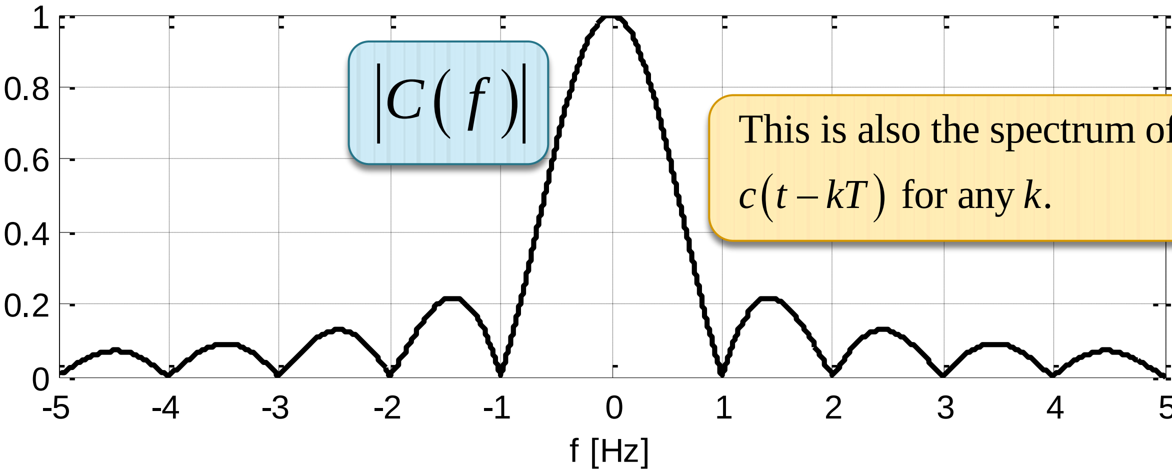
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT)$$



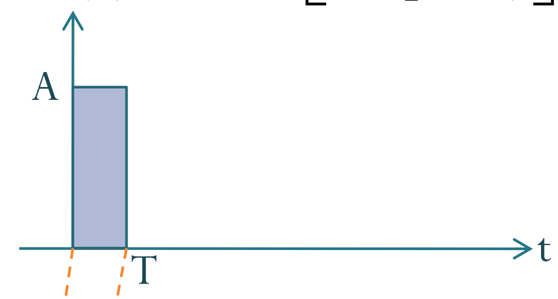
Spectrum of Digital Data (3/4)

$(A = 1, T = 1)$

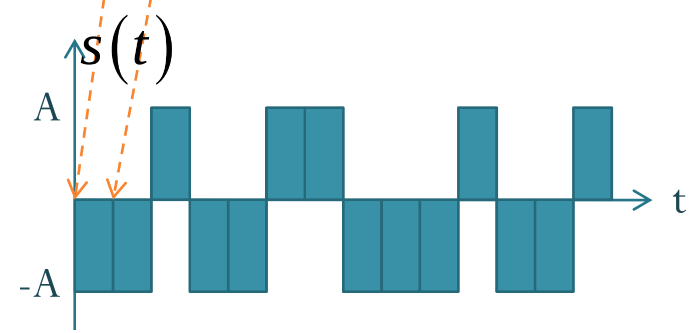


This is also the spectrum of $c(t - kT)$ for any k.

$$c(t) = A \times 1\left[t \in [0, T)\right]$$



m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

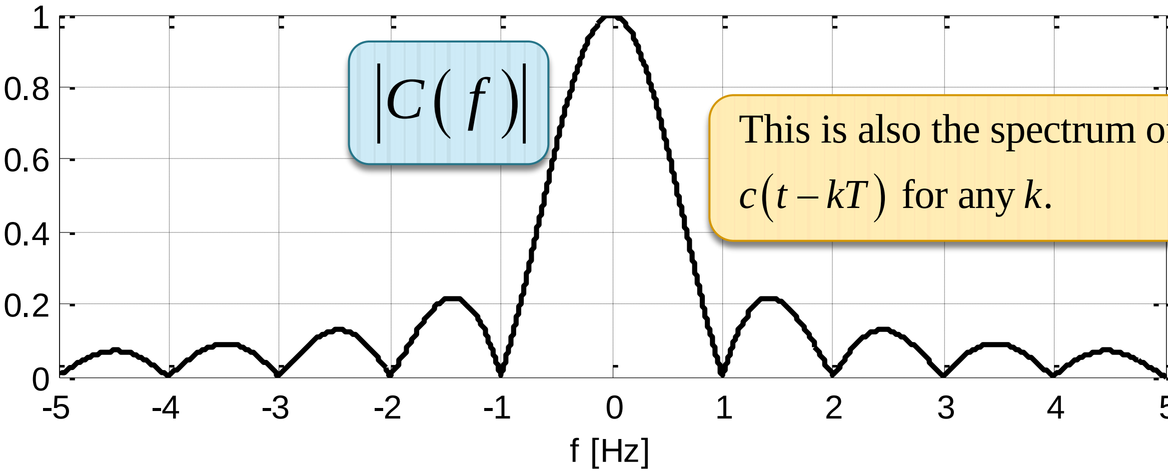


$$s(t) = \sum_{k=0}^{n-1} m_k c(t - kT)$$

$$\xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{n-1} m_k e^{-j2\pi fkT}$$

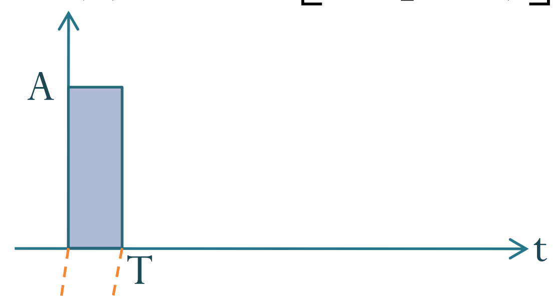
Spectrum of Digital Data (4/4)

$(A=1, T=1)$

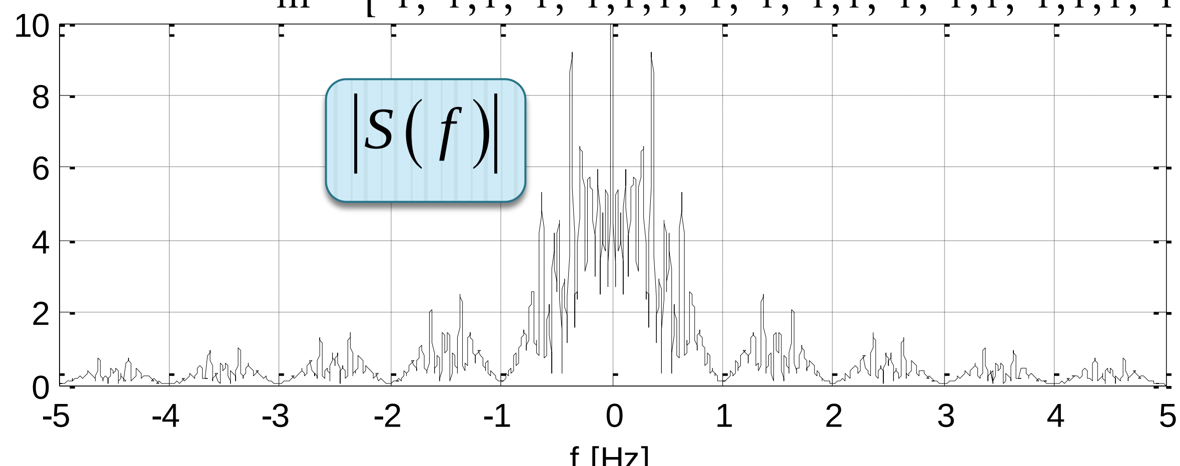


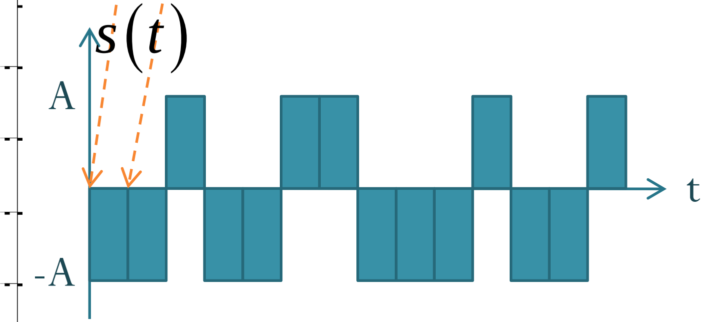
This is also the spectrum of $c(t-kT)$ for any k.

$$c(t) = A \times 1\left[t \in [0,T)\right]$$



m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]





$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT) \xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{n-1} m_k e^{-j2\pi fkT}$$

- In words, for any result or relationship between $g(t)$ and $G(f)$, there exists a dual result or relationship, obtained by interchanging the roles of $g(t)$ and $G(f)$ in the original result (along with some minor modifications arising because of a sign change).

  In particular, if the Fourier transform of $g(t)$ is $G(f)$, then the Fourier transform of $G(f)$ with f replaced by t is the original time-domain signal with t replaced by $-f$.

- If we use the ω-definition (5), we get a similar relationship with an extra factor of 2π:

$$G_2(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} 2\pi g(-\omega).$$

Example 2.19. $x(t) = \cos(2\pi a f_0 t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}\left(\delta(f - af_0) + \delta(f + af_0)\right).$

$\cos(2\pi f_0 t) \xrightarrow{\mathcal{F}} \frac{1}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right)$

$\cos(2\pi a f_0 t) = \cos(2\pi f_0 (at)) \longrightarrow \frac{1}{|a|}\left(\frac{1}{2}\left(\delta\left(\frac{f}{a} - f_0\right) + \delta\left(\frac{f}{a} + f_0\right)\right)\right)$

(scaling theorem)

$= \frac{1}{2}\left(\delta(f - af_0) + \delta(f + af_0)\right)$

$\uparrow\ \delta(at) = \frac{1}{|a|}\delta(t)$

Example 2.20. From Example 2.8, we know that

$$1\left[|t| \le a\right] \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} 2a\ \operatorname{sinc}\left(2\pi a f\right) \tag{15}$$

By the duality theorem, we have

$$2a\operatorname{sinc}(2\pi a t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} 1[|-f| \le a],$$

which is the same as

$$\operatorname{sinc}(2\pi f_0 t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2f_0} 1[|f| \le f_0]. \tag{16}$$

Both transform pairs are illustrated in Figure 2.

Example 2.21. Let's try to derive the time-shift property from the frequency-shift property. We start with an arbitrary function $g(t)$. Next we will define another function $x(t)$ by setting $X(f)$ to be $g(f)$. Note that f here is just a dummy variable; we can also write $X(t) = g(t)$. Applying the duality theorem to the transform pair $x(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X(f)$, we get another transform pair $X(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} x(-f)$. The LHS is $g(t)$; therefore, the RHS must be $G(f)$. This implies $G(f) = x(-f)$. Next, recall the frequency-shift property:

$$e^{j2\pi ct}x(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X(f-c).$$

The duality theorem then gives

$$X(t-c) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} e^{j2\pi c-f}x(-f).$$

Replacing $X(t)$ by $g(t)$ and $x(-f)$ by $G(f)$, we finally get the time-shift property.

Definition 2.22. The **convolution** of two signals, $x_1(t)$ and $x_2(t)$, is a new function of time, $x(t)$. We write

$$x = x_1 * x_2.$$

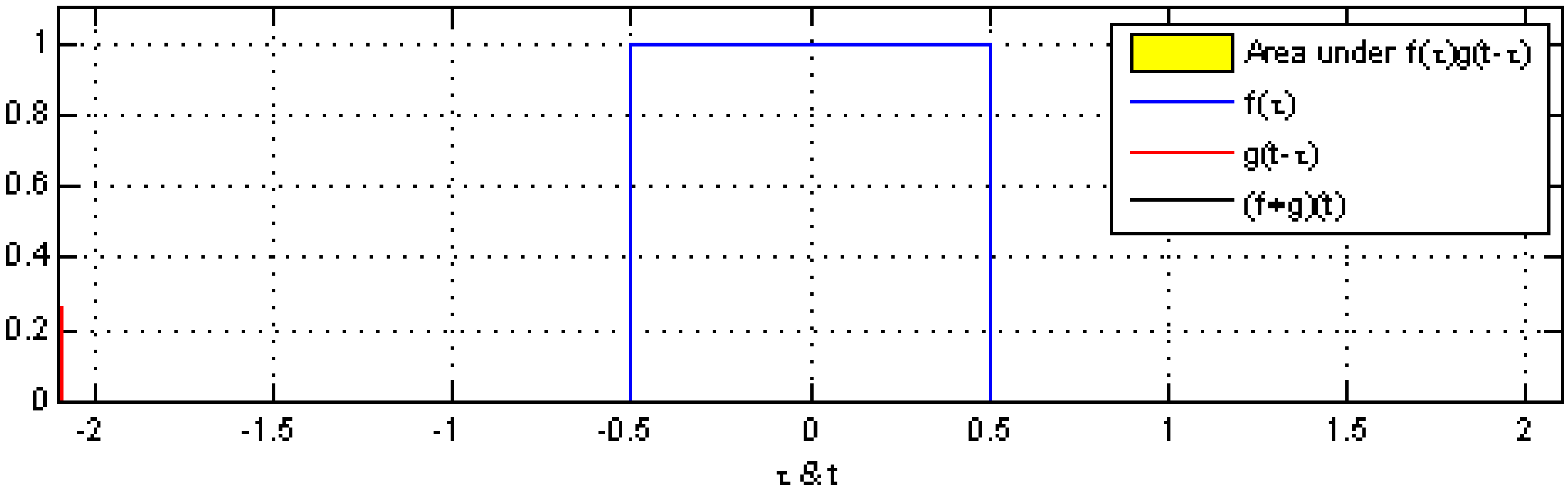
It is defined as the integral of the product of the two functions after one is reversed and shifted:

$$x(t) = (x_1 * x_2)(t) \tag{17}$$

$$= \int_{-\infty}^{+\infty} x_1(\mu)x_2(t-\mu)d\mu = \int_{-\infty}^{+\infty} x_1(t-\mu)x_2(\mu)d\mu. \tag{18}$$

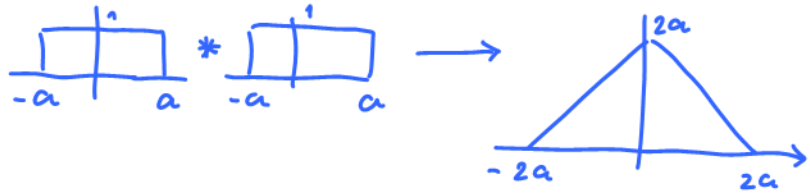
- Note that t is a parameter as far as the integration is concerned.
- The integrand is formed from x_1 and x_2 by three operations:
  - (a) time reversal to obtain $x_2(-\mu)$,
  - (b) time shifting to obtain $x_2(-(\mu - t)) = x_2(t-\mu)$, and
  - (c) multiplication of $x_1(\mu)$ and $x_2(t-\mu)$ to form the integrand.
- In some references, (17) is expressed as $x(t) = x_1(t) * x_2(t)$.

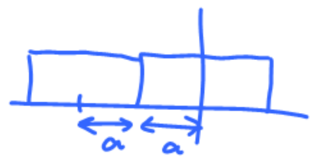
Example: Convolution



Example 2.23. We can get a triangle from convolution of two rectangular waves. In particular,

$$1[|t| \le a] * 1[|t| \le a] = (2a - |t|) \times 1[|t| \le 2a].$$





2.24. Convolution theorem:

(a) Convolution-in-time rule:

$$x_1 * x_2 \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_1 \times X_2. \tag{19}$$

(b) Convolution-in-frequency rule:

$$x_1 \times x_2 \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_1 * X_2. \tag{20}$$

Example 2.25. We can use the convolution theorem to "prove" the frequency-sift property in 2.17.

2.26. From the convolution theorem, we have

- $g^2 \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G * G$
- if g is band-limited to B, then g^2 is band-limited to $2B$

  $|G(f)| = 0$ for $|f| > B$

2.27. Parseval's theorem (Rayleigh's energy theorem, Plancherel formula) for Fourier transform:

$$\int_{-\infty}^{+\infty} |g(t)|^2 dt = \int_{-\infty}^{+\infty} |G(f)|^2 df. \tag{21}$$

The LHS of (21) is called the (total) **energy** of $g(t)$. On the RHS, $|G(f)|^2$ is called the energy spectral density of $g(t)$. By integrating the energy spectral density over all frequency, we obtain the signal 's total energy. The energy contained in the frequency band B can be found from the integral $\int_B |G(f)|^2 df$.

More generally, Fourier transform preserves the inner product [1, Theorem 2.12]:

$$\langle g_1, g_2 \rangle = \int_{-\infty}^{\infty} g_1(t) g_2^*(t) dt = \int_{-\infty}^{\infty} G_1(f) G_2^*(f) df = \langle G_1, G_2 \rangle.$$

2.28. (Heisenberg) **Uncertainty Principle** [1, 6]: Suppose g is a function which satisfies the normalizing condition $\|g\|_2^2 = \int |g(t)|^2 dt = 1$ which automatically implies that $\|G\|_2^2 = \int |G(f)|^2 df = 1$. Then

$$\left(\int t^2 |g(t)|^2 dt\right)\left(\int f^2 |G(f)|^2 df\right) \geq \frac{1}{16\pi^2}, \tag{22}$$

and equality holds if and only if $g(t) = Ae^{-Bt^2}$ where $B > 0$ and $|A|^2 = \sqrt{2B/\pi}$.

- In fact, we have

  $$\left(\int t^2 |g(t-t_0)|^2 dt\right)\left(\int f^2 |G(f-f_0)|^2 df\right) \geq \frac{1}{16\pi^2},$$

  for every t_0, f_0.

- The proof relies on Cauchy-Schwarz inequality.

- For any function h, define its dispersion Δ_h as $\frac{\int t^2 |h(t)|^2 dt}{\int |h(t)|^2 dt}$. Then, we can apply (22) to the function $g(t) = h(t)/\|h\|_2$ and get

  $$\Delta_h \times \Delta_H \geq \frac{1}{16\pi^2}.$$

2.29. A signal cannot be simultaneously time-limited and band-limited.

Proof. Suppose $g(t)$ is simultaneously (1) time-limited to T_0 and (2) band-limited to B. Pick any positive number T_s and positive integer K such that $f_s = \frac{1}{T_s} > 2B$ and $K > \frac{T_0}{T_s}$. The sampled signal $g_{T_s}(t)$ is given by

$$g_{T_s}(t) = \sum_k g[k]\delta\left(t - kT_s\right) = \sum_{k=-K}^{K} g[k]\delta\left(t - kT_s\right)$$

where $g[k] = g\left(kT_s\right)$. Now, because we sample the signal faster than the Nyquist rate, we can reconstruct the signal g by producing $g_{T_s} * h_r$ where the LPF h_r is given by

$$H_r(\omega) = T_s 1[\omega < 2\pi f_c]$$

with the restriction that $B < f_c < \frac{1}{T_s} - B$. In frequency domain, we have

$$G(\omega) = \sum_{k=-K}^{K} g[k]e^{-jk\omega T_s} H_r(\omega).$$

Consider ω inside the interval $I = (2\pi B, 2\pi f_c)$. Then,

$$0 \overset{\omega>2\pi B}{=} G(\omega) \overset{\omega<2\pi f_c}{=} T_s \sum_{k=-K}^{K} g\left(kT_s\right) e^{-jk\omega T_s} \overset{z=e^{j\omega T_s}}{=} T_s \sum_{k=-K}^{K} g\left(kT_s\right) z^{-k} \tag{23}$$

Because $z \neq 0$, we can divide (23) by z^{-K} and then the last term becomes a polynomial of the form

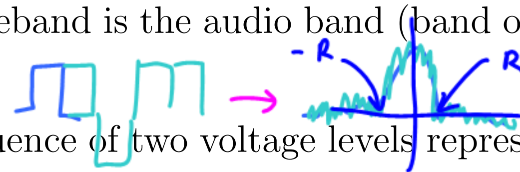
$$a_{2K}z^{2K} + a_{2K-1}z^{2K-1} + \cdots + a_1 z + a_0.$$

By fundamental theorem of algebra, this polynomial has only finitely many roots– that is there are only finitely many values of $z = e^{j\omega T_s}$ which satisfies (23). Because there are uncountably many values of ω in the interval I and hence uncountably many values of $z = e^{j\omega T_s}$ which satisfy (23), we have a contradiction. □

3 Modulation and Frequency Shifting

Definition 3.1. The term **baseband** is used to designate the band of frequencies of the signal delivered by the source.

Example 3.2. In telephony, the baseband is the audio band (band of voice signals) of 0 to 3.5 kHz.



Example 3.3. For digital data (sequence of two voltage levels representing 0 and 1) at a rate of R bits per second, the baseband is 0 to R Hz.

Definition 3.4. **Modulation** is a process that causes a shift in the range of frequencies in a signal.

- The modulation process commonly translates an information-bearing signal to a new spectral location depending upon the intended frequency for transmission.

Definition 3.5. In **baseband communication**, baseband signals are transmitted without modulation, that is, without any shift in the range of frequencies of the signal.

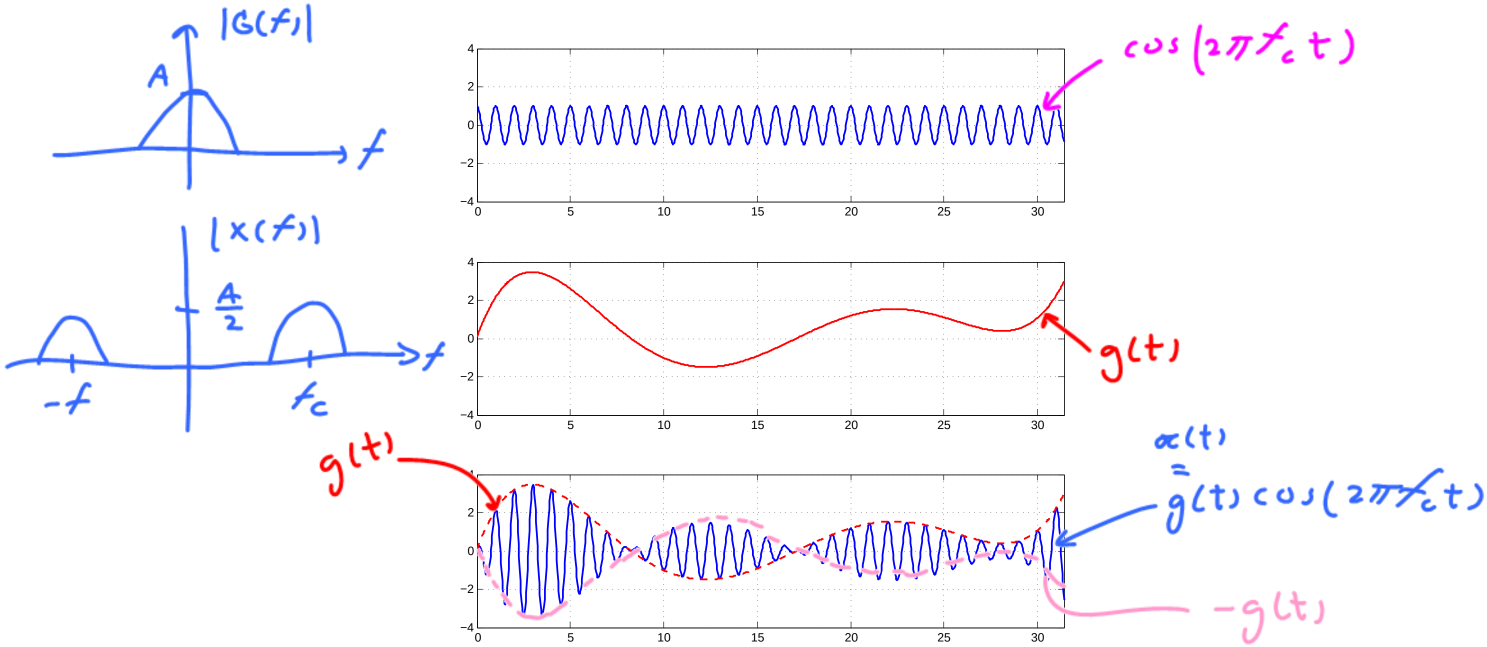
3.6. Recall the frequency-shift property:

$$e^{j2\pi f_c t} g(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G(f - f_c).$$

This property states that multiplication of a signal by a factor $e^{j2\pi f_c t}$ shifts the spectrum of that signal by $f = f_c$.

3.7. Frequency-shifting (frequency translation) in practice is achieved by multiplying $g(t)$ by a sinusoidal:

$$g(t)\cos(2\pi f_c t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}\left(G(f - f_c) + G(f + f_c)\right).$$



Similarly,

$$g(t)\cos(2\pi f_c t + \phi) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}\left(G(f - f_c)e^{j\phi} + G(f + f_c)e^{-j\phi}\right).$$

Definition 3.8. $\cos(2\pi f_c t + \phi)$ is called the (sinusoidal) **carrier signal** and f_c is called the **carrier frequency**. In general, it can also has amplitude A and hence the general expression of the carrier signal is $A\cos(2\pi f_c t + \phi)$.

3.9. Examples of situations where modulation (spectrum shifting) is useful:

(a) **Channel passband matching**: Recall that, for a linear, time-invariant (LTI) system, the input-output relationship is given by

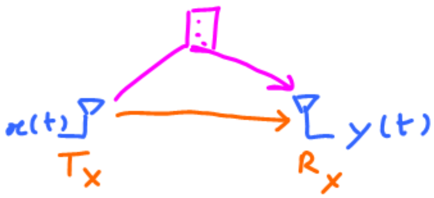
$$y(t) = h(t) * x(t)$$

where $x(t)$ is the input, $y(t)$ is the output, and $h(t)$ is the **impulse response** of the system. In which case,

$$Y(f) = H(f)X(f)$$

where $H(f)$ is called the **transfer function** or **frequency response** of the system. $|H(f)|$ and $\angle H(f)$ are called the **amplitude response** and **phase response**, respectively. Their plots as functions of f show at a glance how the system modifies the amplitudes and phases of various sinusoidal inputs.

multipath wireless channel



$y(t) = 0.9\,x(t-5\text{ms}) + 0.5\,x(t-6\text{ms})$

$h(t) = 0.9\,\delta(t-5\text{ms}) + 0.5\,\delta(t-6\text{ms})$

$H(f) = 0.9\,e^{-j2\pi f 5\text{ms}} + 0.5\,e^{-j2\pi f 6\text{ms}}$

$c = f\lambda$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 10^5 = 100$ km

spaceflight altitude starts here. (10-20 km → airplane altitude)

3 GHz → 10 cm
60 GHz → 5 mm
100 MHz → 3 m

(b) **Reasonable antenna size**: For effective radiation of power over a radio link, the antenna size must be on the order of the wavelength of the signal to be radiated.

$f = 3$ kHz

- Audio signal frequencies are so low (wavelengths are so large) that impractically large antennas will be required for radiation. Here, shifting the spectrum to a higher frequency (a smaller wavelength) by modulation solves the problem.

Frequency-division multiple access (FDMA) multi-user

(c) **Frequency-division multiplexing** (FDM):

Transmitting several signals simultaneously over a channel by using different frequency bands.

- If several signals, each occupying the same frequency band, are transmitted simultaneously over the same transmission medium, they will all interfere; it will be difficult to separate or retrieve them at a receiver.
- For example, if all radio stations decide to broadcast audio signals simultaneously, the receiver will not be able to separate them.
- One solution is to use modulation whereby each radio station is assigned a distinct carrier frequency. Each station transmits a modulated signal, thus shifting the signal spectrum to its allocated band,

which is not occupied by any other station. A radio receiver can pick up any station by tuning to the band of the desired station.

Definition 3.10. Communication that uses modulation to shift the frequency spectrum of a signal is known as **carrier communication**. [2, p 151]

3.11. A sinusoidal carrier signal $A\cos(2\pi f_c t + \phi)$ has three basic parameters: amplitude, frequency, and phase. Varying these parameters in proportion to the baseband signal results in amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM), respectively. Collectively, these techniques are called **continuous-wave modulation** in [7, p 111].

We will use $m(t)$ to denote the baseband signal. We will assume that $m(t)$ is band-limited to B; that is, $|M(f)| = 0$ for $|f| > B$. Note that we usually call it the message or the modulating signal.

Definition 3.12. The process of recovering the signal from the modulated signal (retranslating the spectrum to its original position) is referred to as **demodulation**, or **detection**.

Important Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$2\cos^2 x = 1 + \cos(2x)$$

$$2\sin^2 x = 1 - \cos(2x)$$

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

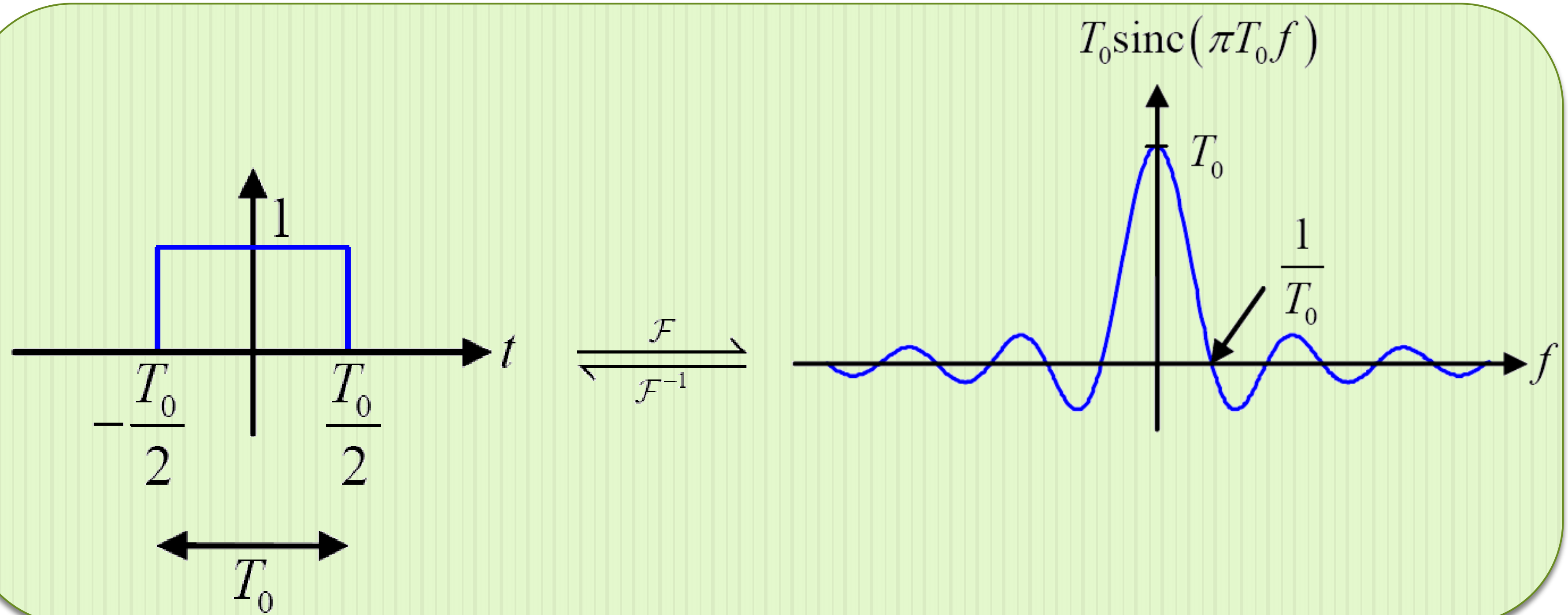
$$\cos(2\pi f_c t + \theta) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}\delta(f - f_c)e^{j\theta} + \frac{1}{2}\delta(f + f_c)e^{-j\theta}$$

$$g(t - t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi ft_0} G(f)$$

$$e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f - f_0)$$

$$m(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}M(f - f_c) + \frac{1}{2}M(f + f_c)$$

Frequency-Domain Analysis



Shifting Properties: $g(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} G(f)$ $\quad e^{j2\pi f_0 t} g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f-f_0)$

Modulation: $m(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c)$

4 Amplitude modulation: DSB-SC and QAM

Definition 4.1. Amplitude modulation is characterized by the fact that the amplitude A of the carrier $A\cos(2\pi f_c t + \phi)$ is varied in proportion to the baseband (message) signal $m(t)$.

- Because the amplitude is time-varying, we may write the modulated carrier as
$$A(t)\cos(2\pi f_c t + \phi)$$
- Because the amplitude is linearly related to the message signal, this technique is also called **linear modulation**.

4.1 Double-sideband suppressed carrier (DSB-SC) modulation

4.2. Basic idea:

$$\text{LPF}\left\{\underbrace{\left(m\left(t\right) \times \sqrt{2}\cos\left(2\pi f_c t\right)\right)}_{x(t)} \times \left(\sqrt{2}\cos\left(2\pi f_c t\right)\right)\right\} = m\left(t\right). \qquad (24)$$

$$\begin{aligned} x\left(t\right) &= m\left(t\right) \times \sqrt{2}\cos\left(2\pi f_c t\right) = \sqrt{2} m\left(t\right)\cos\left(2\pi f_c t\right) \\ X\left(f\right) &= \sqrt{2}\left(\frac{1}{2}\left(M\left(f-f_c\right) + M\left(f+f_c\right)\right)\right) \\ &= \frac{1}{\sqrt{2}}\left(M\left(f-f_c\right) + M\left(f+f_c\right)\right) \end{aligned}$$

Similarly,

$$\begin{aligned} v\left(t\right) &= y\left(t\right) \times \sqrt{2}\cos\left(2\pi f_c t\right) = \sqrt{2} x\left(t\right)\cos\left(2\pi f_c t\right) \\ V\left(f\right) &= \frac{1}{\sqrt{2}}\left(X\left(f-f_c\right) + X\left(f+f_c\right)\right) \end{aligned}$$

Alternatively, we can use the trig. identity from Example 2.3:

$$\begin{aligned}
v(t) &= \sqrt{2}x(t)\cos(2\pi f_c t) = \sqrt{2}\left(\sqrt{2}m(t)\cos(2\pi f_c t)\right)\cos(2\pi f_c t) \\
&= 2m(t)\cos^2(2\pi f_c t) = m(t)\left(\cos(2(2\pi f_c t)) + 1\right) \\
&= m(t) + m(t)\cos(2\pi(2f_c)t)
\end{aligned}$$

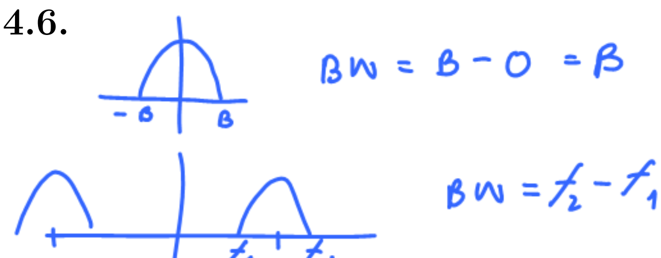
4.3. In the process of modulation, observe that we need $f_c > W$ in order to avoid overlap of the spectra.

4.4. Observe that the modulated signal spectrum centered at f_c, is composed of two parts: a portion that lies above f_c, known as the **upper sideband** (USB), and a portion that lies below f_c, known as the **lower sideband** (LSB). Similarly, the spectrum centered at $-f_c$ has upper and lower sidebands. Hence, this is a modulation scheme with **double sidebands**.

4.2 Quadrature Amplitude Modulation (QAM)

Definition 4.5. One of the possible definition for the ***bandwidth*** (***BW***) of a signal is the difference between the highest frequency and the lowest frequency in the positive-f part of the signal spectrum.

Example 4.6.



4.7. If $g_1(t)$ and $g_2(t)$ have bandwidths B_1 and B_2 Hz, respectively, the bandwidth of $g_1(t)g_2(t)$ is $B_1 + B_2$ Hz.

This result follows from the application of the width property of convolution[3] to the convolution-in-frequency property. This property states that the width of $x * y$ is the sum of the widths of x and y.

Consequently, if the bandwidth of $g(t)$ is B Hz, then the bandwidth of $g^2(t)$ is $2B$ Hz, and the bandwidth of $g^n(t)$ is nB Hz.

[3]The width property of convolution does not hold in some pathological cases. See [2, p 98].

4.8. Recall that for real-valued baseband signal $m(t)$, the conjugate symmetry property from 2.16 says that

$$M(-f) = (M(f))^*.$$

The DSB spectrum has two sidebands: the upper sideband (USB) and the lower sideband (LSB), both containing complete information about the baseband signal $m(t)$. As a result, DSB signals occupy twice the bandwidth required for the baseband. To improve the spectral efficiency of amplitude modulation, there exist two basic schemes to either utilize or remove the spectral redundancy:

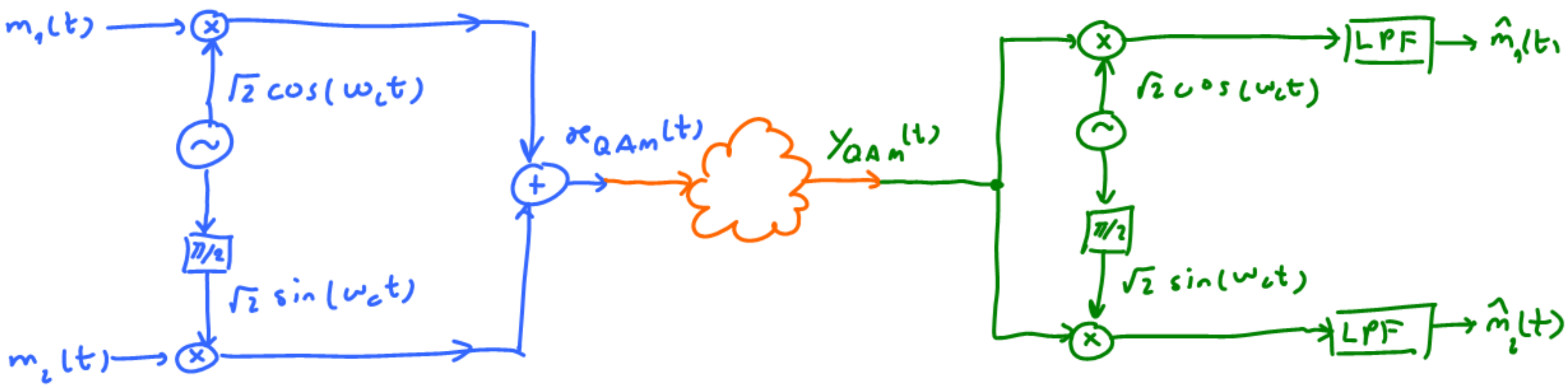
(a) Single-sideband (SSB) modulation, which removes either the LSB or the USB so that for one message signal $m(t)$, there is only a bandwidth of B Hz.

(b) Quadrature amplitude modulation (QAM), which utilizes spectral redundancy by sending two messages over the same bandwidth of $2B$ Hz.

We will only discussed QAM here. SSB discussion can be found in [7, Section 3.1.3] and [2, Section 4.5].

Definition 4.9. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the corresponding QAM signal:

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$



- QAM operates by transmitting two DSB signals via carriers of the same frequency but in phase quadrature.

1 cycle

$\frac{1}{4} \Rightarrow \frac{1}{4}\times 2\pi = \frac{\pi}{2}$

- QAM can be exactly generated without requiring sharp cutoff bandpass filters.
- Both modulated signals simultaneously occupy the same frequency band.
- The upper channel is also known as the ***in-phase*** (***I***) channel and the lower channel is the ***quadrature*** (***Q***) channel.

4.10. ***Demodulation***: The two baseband signals can be separated at the receiver by synchronous detection:

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos(\omega_c t)\right\} = m_1(t)$$
$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin(\omega_c t)\right\} = m_2(t)$$

- $m_1(t)$ and $m_2(t)$ can be separately demodulated.

4.11. Sinusoidal form:

$$x_{\mathrm{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \theta(t)),$$

where

$$E(t) = \sqrt{m_1^2(t) + m_2^2(t)}$$
$$\theta(t) = + \tan^{-1}\left(\frac{m_2(t)}{m_1(t)}\right)$$

4.12. Complex form:

$$x_{\mathrm{QAM}}(t) = \sqrt{2}\mathrm{Re}\left\{(m(t))\, e^{j2\pi f_c t}\right\}$$

where $m(t) = m_1(t) - jm_2(t)$.

$(m_1(t) - j\, m_2(t))(\cos(2\pi f_c t) + j\sin(2\pi f_c t))$

- If we use $-\sin(\omega_c t)$ instead of $\sin(\omega_c t)$,

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) - m_2(t)\sqrt{2}\sin(\omega_c t)$$

  and

$$m(t) = m_1(t) + jm_2(t).$$

- We refer to $m(t)$ as the ***complex envelope*** (or ***complex baseband signal***) and the signals $m_1(t)$ and $m_2(t)$ are known as the ***in-phase*** and ***quadrature(-phase)*** components of $x_{\text{QAM}}(t)$.

- The term "quadrature component" refers to the fact that it is in phase quadrature ($\pi/2$ out of phase) with respect to the in-phase component.

- Key equation:

$$\text{LPF}\left\{\underbrace{\left(\text{Re}\left\{m(t) \times \sqrt{2}e^{j2\pi f_c t}\right\}\right)}_{x(t)} \times \left(\sqrt{2}e^{-j2\pi f_c t}\right)\right\} = m(t).$$

4.13. Three equivalent ways of saying exactly the same thing:

(a) the complex-valued envelope $m(t)$ complex-modulates the complex carrier $e^{j2\pi f_c t}$,

(b) the real-valued amplitude $E(t)$ and phase $\theta(t)$ real-modulate the amplitude and phase of the real carrier $\cos(\omega_c t)$,

(c) the in-phase signal $m_1(t)$ and quadrature signal $m_2(t)$ real-modulate the real in-phase carrier $\cos(\omega_c t)$ and the real quadrature carrier $\sin(\omega_c t)$.

4.14. References: [7, Sect. 2.9.4], [2, Sect. 4.4], and [4, Sect. 1.4.1]

4.15. Question: In engineering and applied science, measured signals are real. Why should real measurable effects be represented by complex signals?

Answer: One complex signal (or channel) can carry information about two real signals (or two real channels), and the algebra and geometry of analyzing these two real signals as if they were one complex signal brings economies and insights that would not otherwise emerge.

QAM

$$s(t) = \overbrace{m_I(t)}^{\text{In-phase component}}\cos(\omega_c t) - \overbrace{m_Q(t)}^{\text{Quadrature component}}\sin(\omega_c t)$$

$$= \mathrm{Re}\left\{\underbrace{\left(m_I(t) + jm_Q(t)\right)}_{m(t)} e^{j\omega_c t}\right\}$$

- ❑ Complex baseband signal
- ❑ Complex envelope of $s(t)$
- ❑ Compex lowpass equivalent signal of $s(t)$

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Instantaneous Frequency (Ex 1/6)

- Suppose you want the frequency of

$$\cos\left(2\pi ft\right)$$

  to change as a function of time $f\left(t\right)=t^2$

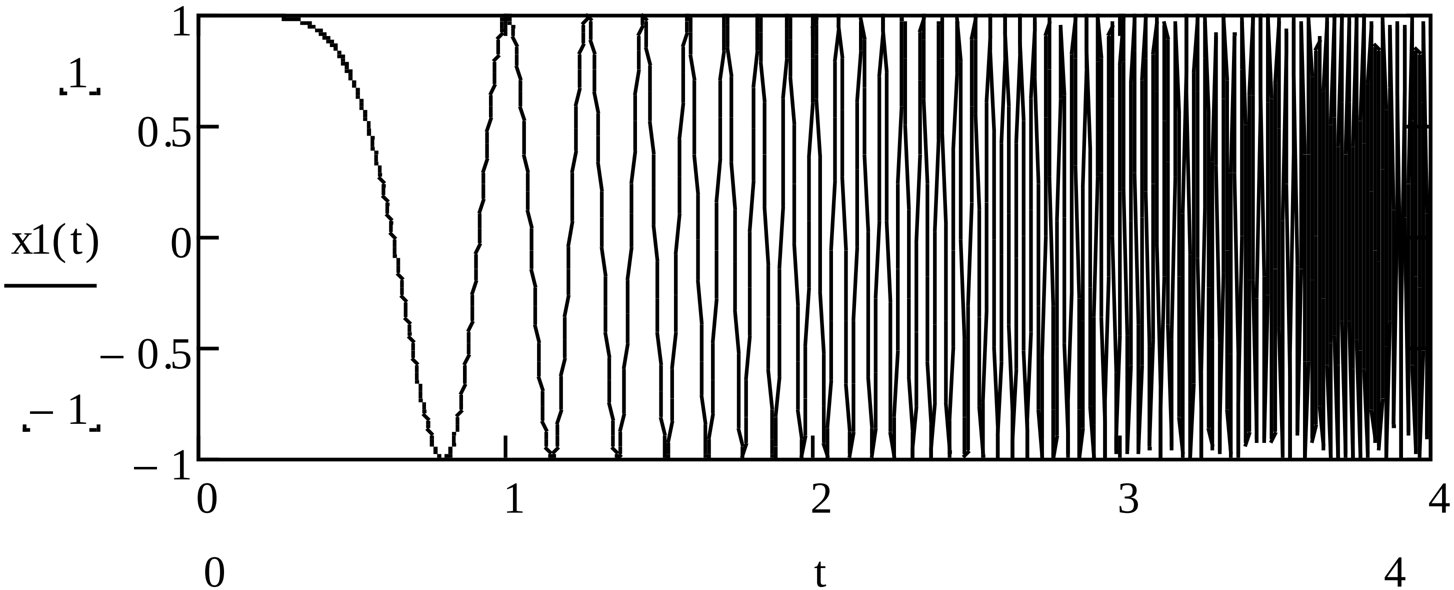
- Intuitively, the following substitution makes sense:

$$\cos\left(2\pi\left(t^2\right)t\right)$$

- But will it work?

Instantaneous Frequency (Ex 2/6)

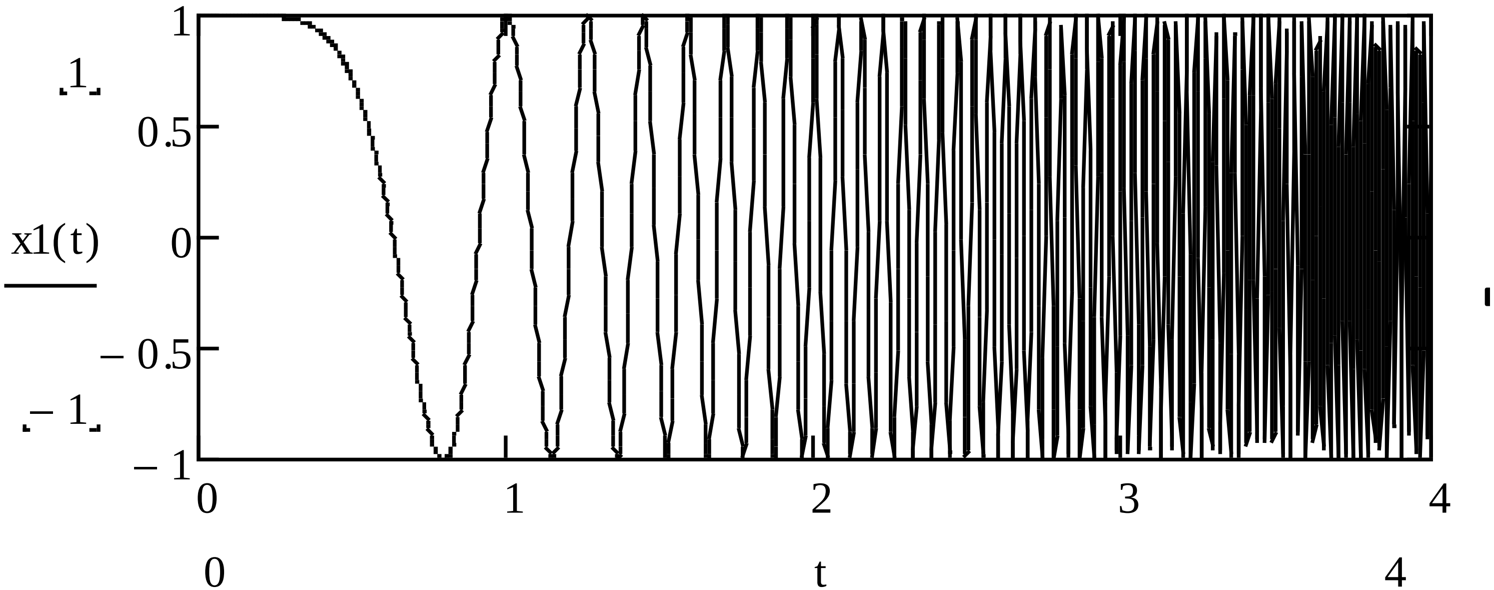
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



At t = 2, frequency = ?

Instantaneous Frequency (Ex 3/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

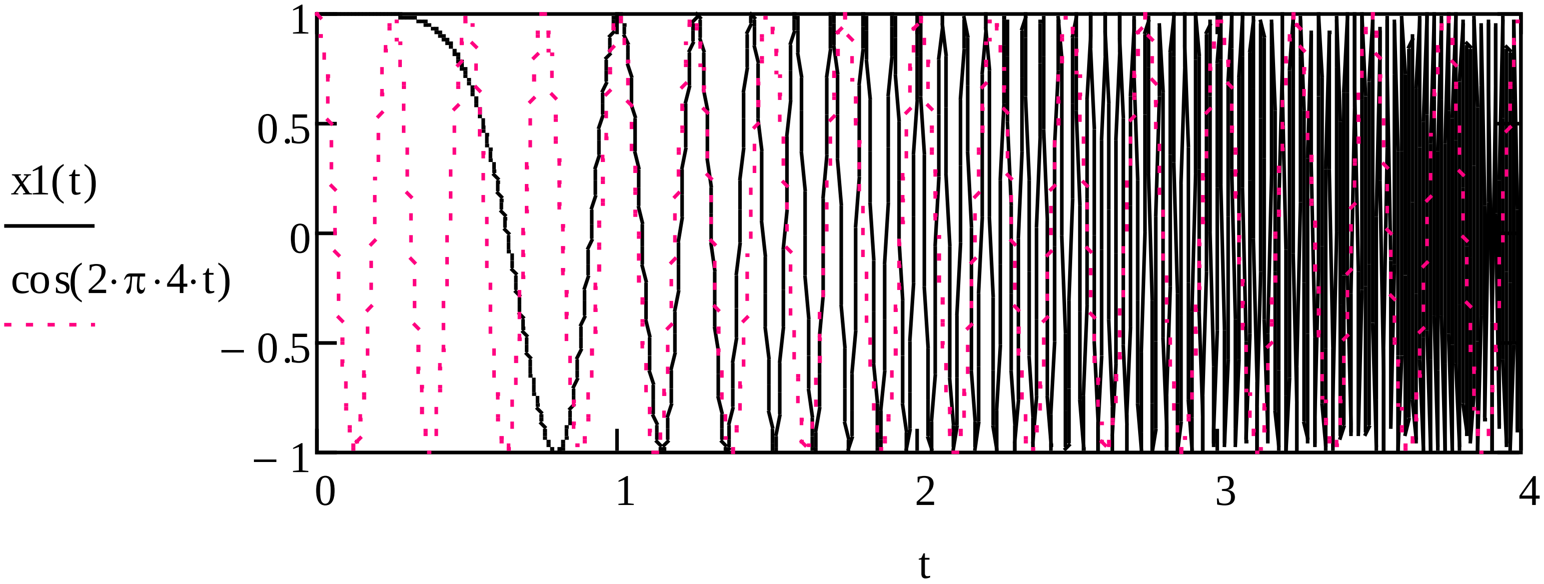


$\cos(2\pi ft)$ → At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency (Ex 4/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

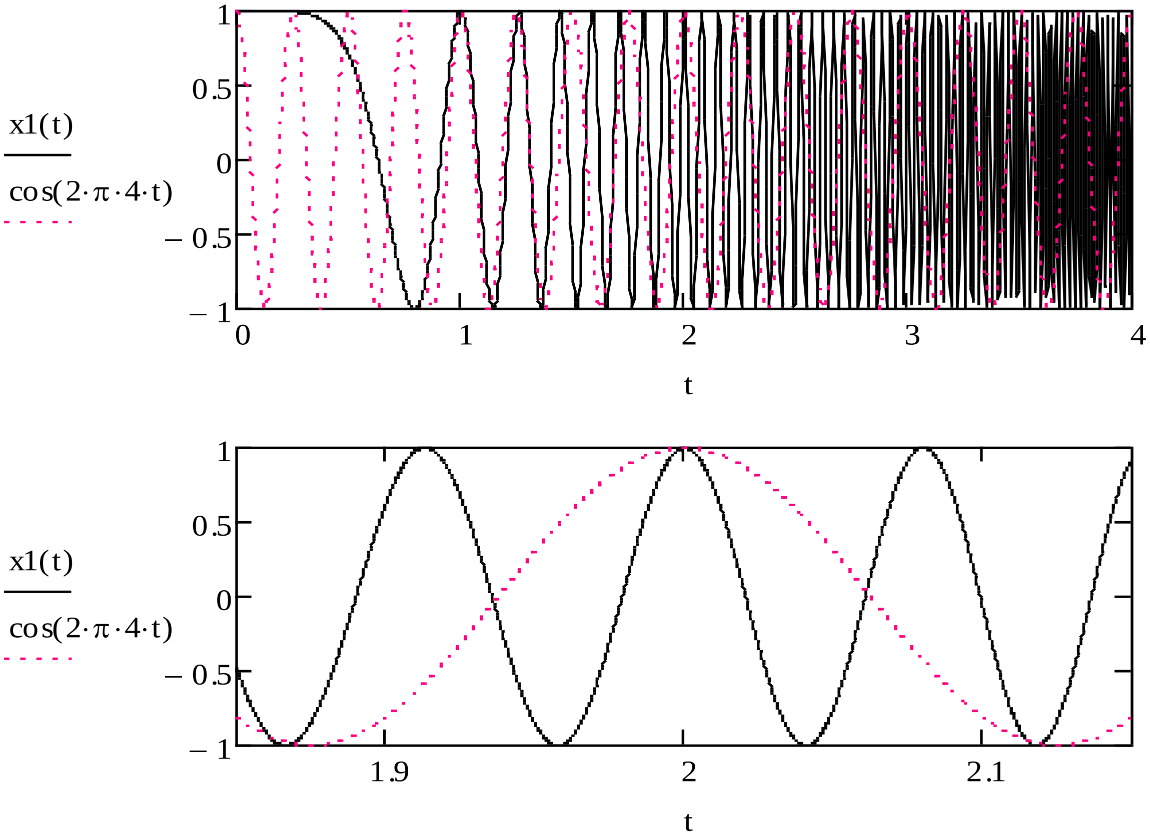


$\cos\left(2\pi ft\right)$ → At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frequency (Ex 5/6)

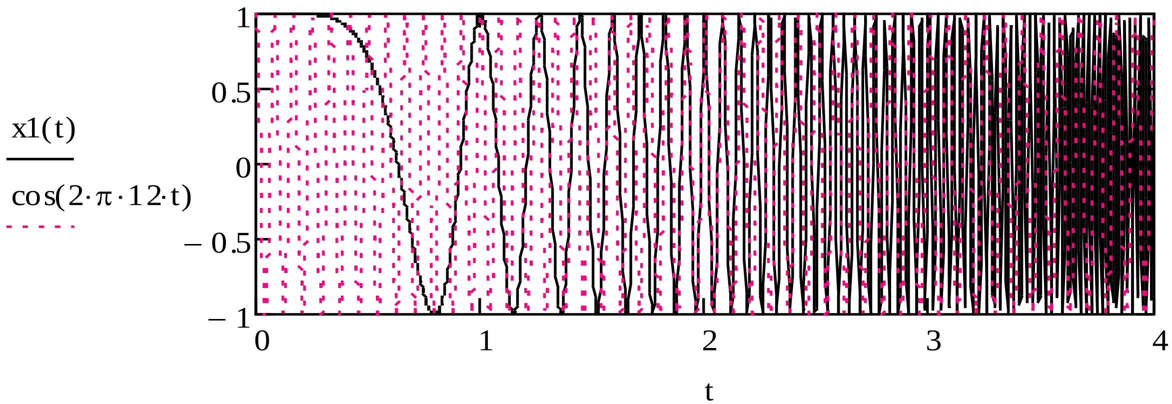
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

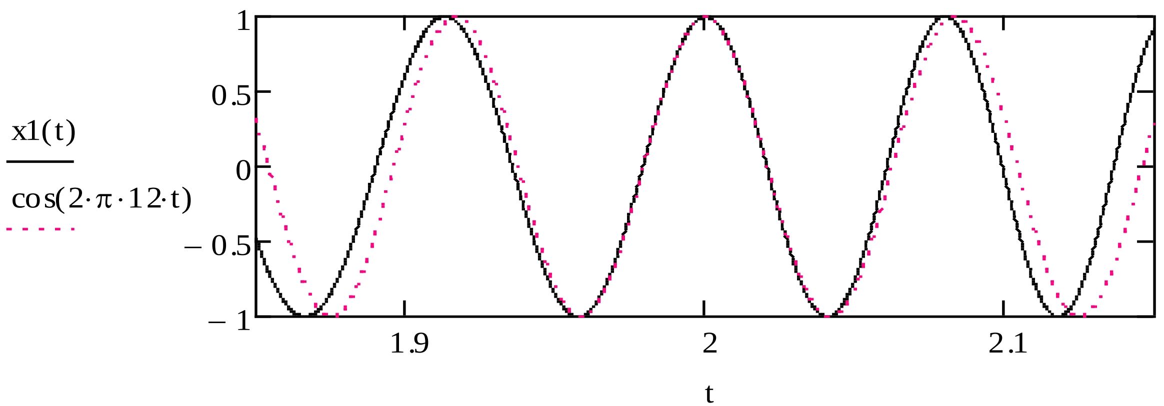


At t = 2, $\cos\left(2\pi\left(t^2\right)t\right)$ oscillates much faster than 4Hz.

Instantaneous Frequency (Ex 6/6)

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$





At t = 2, the frequency of $\cos\left(2\pi\left(t^2\right)t\right)$ is closer to 12 Hz!?

Instantaneous Frequency

of Generalized Sinusoids $x(t) = A\cos(\theta(t))$

$$f(t) = \frac{1}{2\pi}\theta'(t)$$

1-2 Instantaneous Freq

Thursday, January 19, 2012
2:08 PM

Instantaneous frequency

$$x(t) = A\cos(\theta(t))$$

↳ generalized angle

Q: what is the freq. of $x(t)$ around $t = t_0$?

Observation: If $\theta(t) = 2\pi f_0 t + \theta$

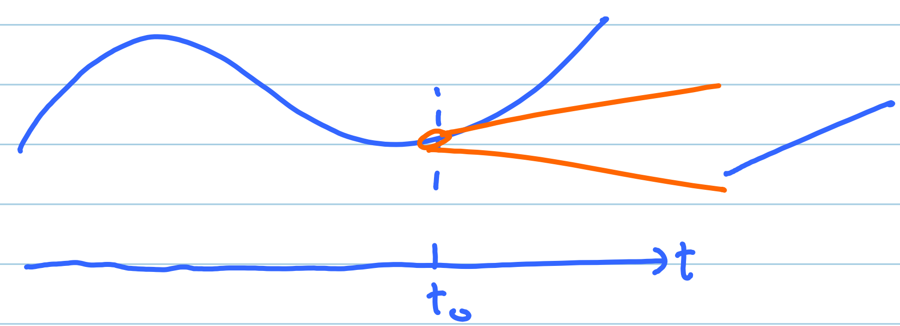
$x(t) = A\cos(2\pi f_0 t + \theta)$

then

$f(t) = f_0 \quad \forall t$

Now, Let's consider general $\theta(t)$.

Plot $\theta(t)$



When we only focus on small interval around t_0,

$\theta(t) \approx mt + c = \theta'(t_0)t + C$

↑ $\theta'(t_0)$

Near $t = t_0$

$$\cos(\theta(t)) \approx \cos(\theta'(t_0)\,t + c)$$

$$\cos(\theta(t)) \approx \cos(\underbrace{\theta'(t_0)}_{2\pi f_0} t + c)$$

$$\rightarrow f_0 = \frac{1}{2\pi}\theta'(t_0)$$

ECS 455 Chapter 1

Introduction & Review

1.3 Wireless Channel (Part 1)

Radio

Severe challange for reliable high speed communication.

- noise
- interference

> time-varying

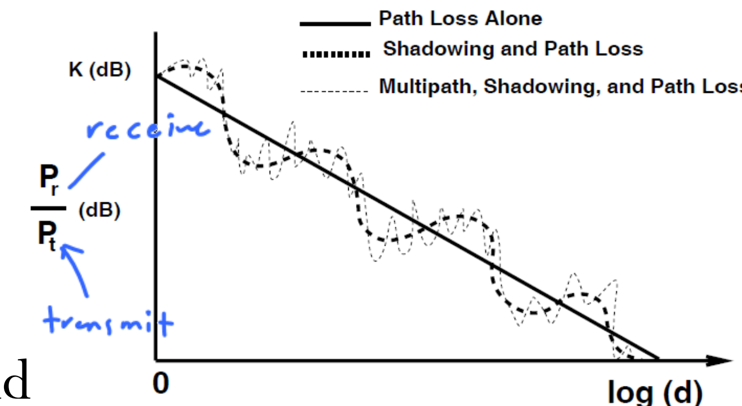
Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

Wireless Channel

Section 1.3 focuses on

1. - **Large-scale** propagation effects
   - 1.1 Path loss
   - 1.2 Shadowing
   - Typically frequency independent
2. - **Small-scale** propagation effects
   - Variation due to the constructive and destructive addition of **multipath** signal components.
   - Occur over very short distances, on the order of the signal wavelength.

(2) will be discussed after midterm



[Goldsmith, 2005, Fig 2.1]

distance

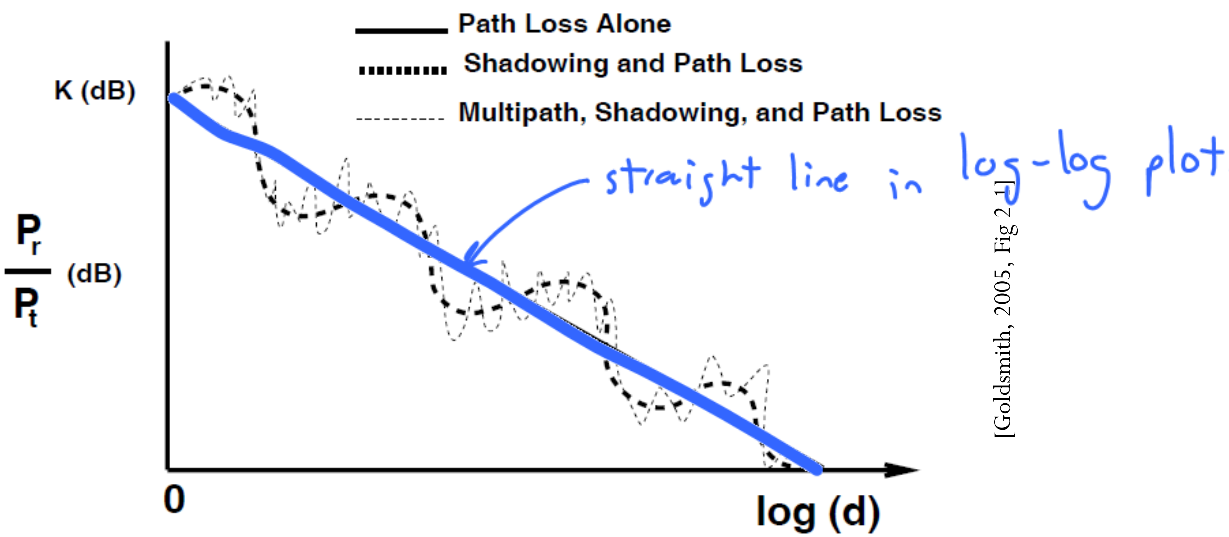
$$\lambda = \frac{c}{f}$$

$c \approx 3\times10^{8}$ [m/s]

$f = 3$ GHz $\rightarrow$ $\lambda = 0.1$ m

(1.1) Path loss

- Caused by
  - dissipation of the power radiated by the transmitter
  - effects of the propagation channel
- Models generally assume that it is the same at a given transmit-receive distance.
  (Need to move over large distance to observe its effect.)
- Variation occurs over **large distances** (100-1000 m)



[Goldsmith, 2005, Fig 2.1]

(1.1) Path Loss

To find Power P in dB, take $10 \log_{10} P$

$$P_L = \frac{\text{Transmitted power}}{\text{Average received power}} = \frac{P_t}{P_r}$$

$P_L[dB] = P_t[dB] - P_r[dB]$

Path Gain

$P_G = \frac{1}{P_L}$

$P_G[dB] = -P_L[dB]$

Averaged over any random variations due to shadowing

- **Free-Space Path Loss:**

$$\frac{P_r}{P_t} \propto \frac{1}{d^2}$$

$P_r = \text{some constant} \times \frac{1}{d^2} \times P_t$

  - P_r falls off inversely proportional to the square of the distance d between the Tx and Rx antennas.
  - For other signal propagation models, P_r falls off more quickly relative to d.

- **Simplified** Path Loss Model:

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma} = \frac{K(d_0)^{\gamma}}{d^{\gamma}}$$

Loss [dB]

$10\log_{10}\frac{P_r}{P_t} = 10\log_{10}(K d_0^{\gamma}) - 10\gamma\log_{10} d$

constant; slope

Friss Equation

1 for non-directional antennas

- One of the most fundamental equations in antenna theory

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}} \lambda}{4\pi d} \right)^2 = \left(\frac{\sqrt{G_{Tx} G_{Rx}} c}{4\pi d f} \right)^2 \propto \frac{1}{d^2 f^2} = \frac{\lambda^2}{d^2}$$

- More power is lost at higher frequencies.

2.4 GH → 5 GHz → 60 GHz

6.4 dB loss $20\log_{10}\frac{5}{2.4}$

21.6 dB loss $20\log_{10}\frac{60}{5}$

- Some of these losses can be offset by reducing the maximum operating range. The remaining loss must be compensated for by increasing the antenna gain.

Path Loss Models

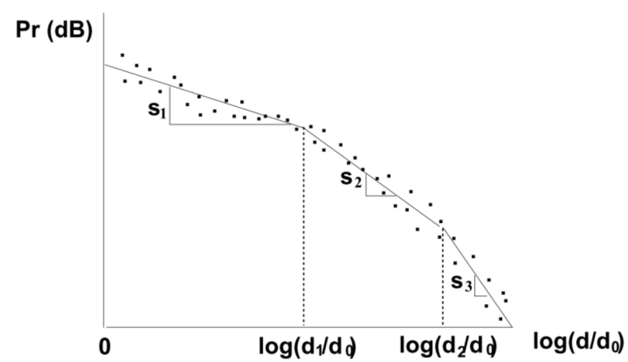
- Analytical models
  - Maxwell's equations
  - Ray tracing

} prohibitive

complex, impractical
Need to know/specify "almost everything" about the environment

- Empirical models
  - Okumura
  - Hata
  - COST 231
    - by EURO-COST (EUROpean Cooperative for Scientific and Technical research)
  - Piecewise Linear (Multi-Slope) Model
- Tradeoff: Simplified Path Loss Model

Developed to predict path loss in typical environment.



Indoor Attenuation Factors

- Building penetration loss: 8-20 dB (better if behind windows)
- Attenuation between floors
  - @ 900 MHz
    - 10-20 dB when the Tx and Rx are separated by a single floor
    - 6-10 dB per floor for the next three subsequent floors
    - A few dB per floor for more than four floors
  - Typically worse at higher frequency.
- Attenuation across floors

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

[Goldsmith, 2005, Sec. 2.5.5]

Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
  - $\left(\frac{\lambda}{4\pi d_0}\right)^2$ for free-space path gain at distance d_0 assuming omnidirectional antennas

(Near-field has scattering phenomena.)

- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.
- γ is the **path loss exponent**.

[Goldsmith, 2005, Table 2.2]

Path Loss Exponent γ

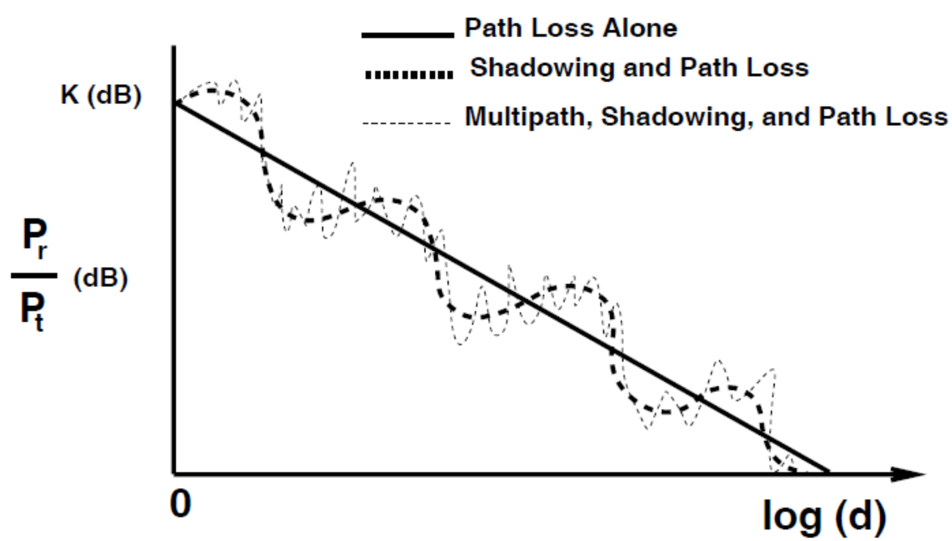
- 2 in free-space model
- 4 in two-ray model
  [Goldsmith, 2005, eq. 2.17]
- Cellular: 3.5 – 4.5
  [Myung and Goodman, 2008 , p 17]
- Larger @ higher freq.
- Lower @ higher antenna heights

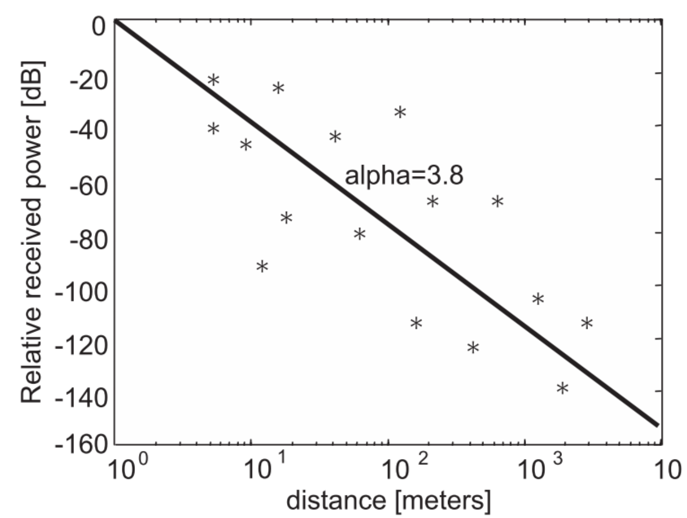
Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

(1.2) Shadowing (or Shadow Fading)

- Caused by **obstacles** (**large objects** such as buildings and hills) between the transmitter and receiver.
  - Think: cloud blocking sunlight
- Attenuate signal power through absorption, reflection, scattering, and diffraction.
- Variation occurs over distances proportional to the length of the obstructing object (**10-100 m** in outdoor environments and less in indoor environments).

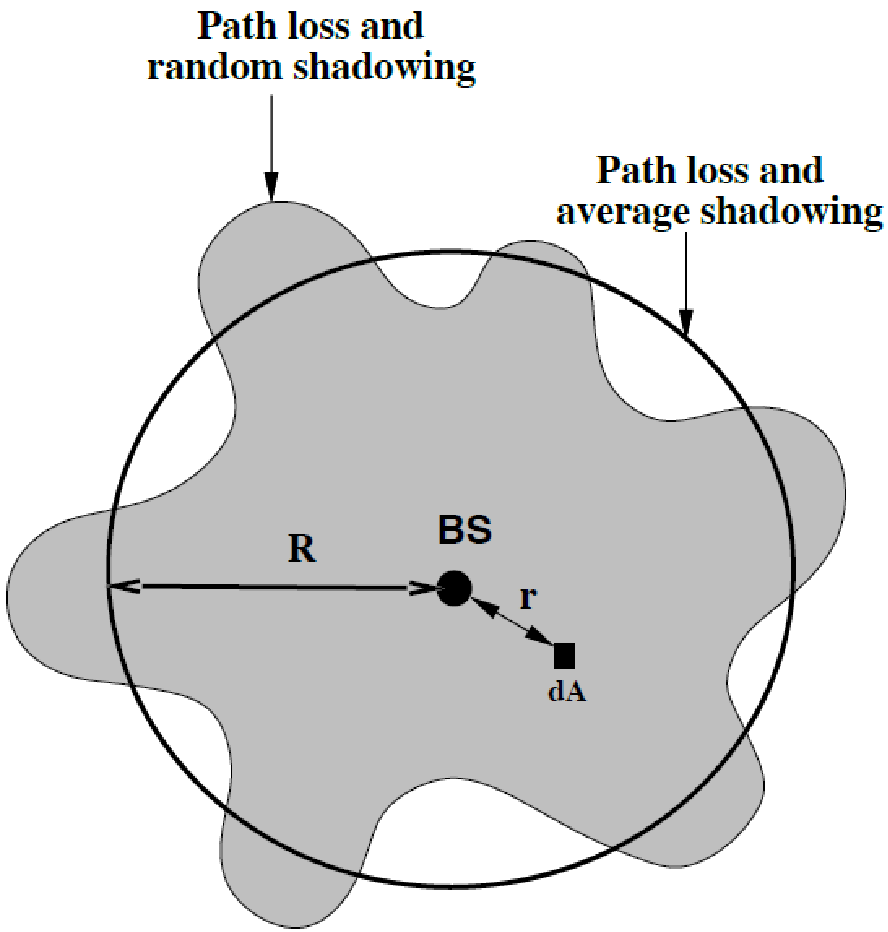
[Goldsmith, 2005, Fig 2.1]





[Myung and Goodman, 2008, Fig 2.1]

Contours of Constant Received Power



[Goldsmith, 2005, Fig 2.10]

$X \rightarrow \square \xrightarrow{F_1 X} \square \xrightarrow{F_2 F_1 X} \square \xrightarrow{F_3 F_2 F_1 X} \square \rightarrow \dots Y = F_1 \times F_2 \times F_3 \times \cdots \times F_N \times X$

$Y[dB] = \sum_i F_i[dB] + X[dB]$, where $\sum_i F_i[dB] \sim \mathcal{N}$

Log-normal shadowing

- Random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects → random variations of the received power at a given distance

$$10\log_{10}\frac{P_t}{P_r} \sim \mathcal{N}\left(\mu,\sigma^2\right)$$

σ: 4 – 13 dB with higher values in urban areas and lower ones in flat rural environments.

μ, σ: in dB

- This model has been confirmed empirically to accurately model the variation in received power in both outdoor and indoor radio propagation environments.

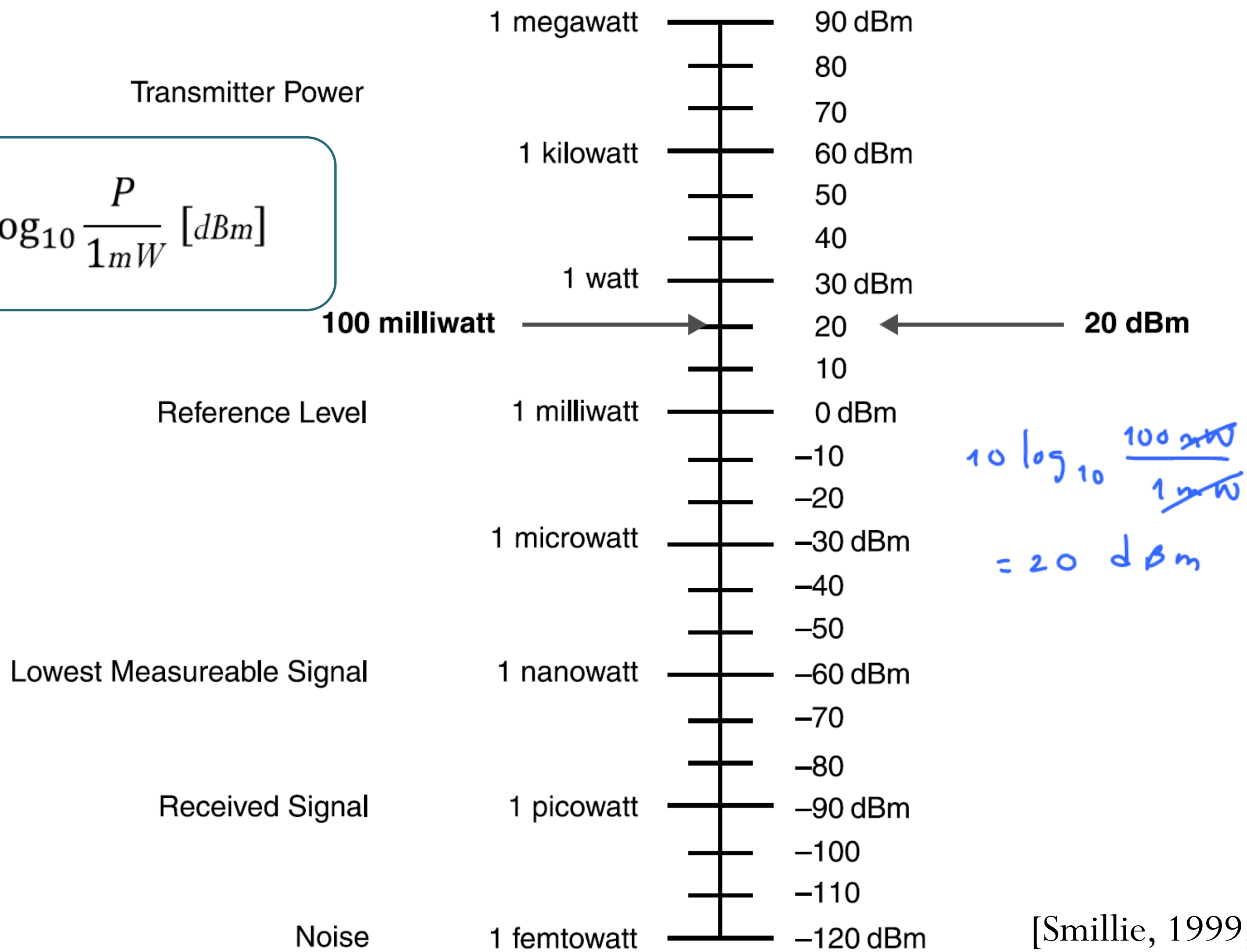
dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
  - hundreds of watts in base station transmitters to
  - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
  - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used

[Scott and Frobenius, 2008, Fig 1.1]

Range of RF Power in Watts and dBm

$$P\,[W] = 10\log_{10}\frac{P}{1mW}\,[dBm]$$



[Smillie, 1999, p 11]

Doppler Shift: 1D Move

- At distance $d = 0$, suppose we have

$$A_0 \cos(2\pi f t + \phi)$$

- At distance r, we have

$\theta(t)$

Time to travel a distance of r

Instantaneous freq.

$$A_r \cos\left(2\pi f\left(t - \frac{r}{c}\right) + \phi\right)$$

$f(t) = \frac{1}{2\pi}\theta'(t) = f - \frac{f}{c} r'(t)$

$= f - \frac{1}{\lambda} r'(t)$

$A_r \cos\left(2\pi f\left(t - \frac{r(t)}{c}\right) + \phi\right)$

- If moving, r becomes $r(t)$.
- If moving ***away*** at a constant velocity v, then $r(t) = r_0 + vt$.

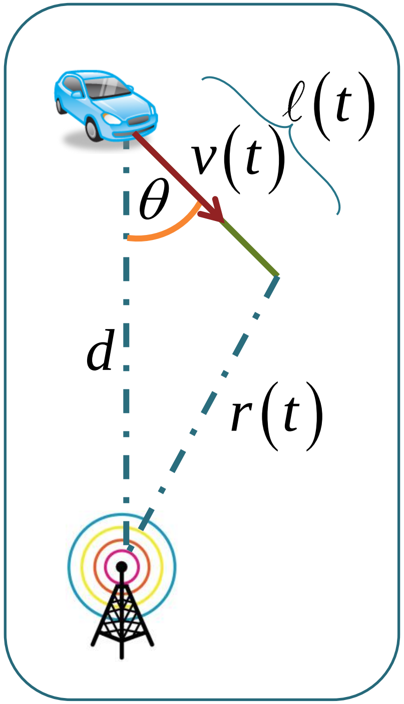
$$A_{r(t)} \cos\left(2\pi f\left(t - \frac{r_0 + vt}{c}\right) + \phi\right) = A_{r(t)} \cos\left(2\pi\left(f - f\frac{v}{c}\right)t - 2\pi f\frac{r_0}{c} + \phi\right)$$

Frequency shift

$$\Delta f = \frac{v}{\lambda}$$

Doppler Shift: With angle

Rx speed = $v(t)$. At time t, cover distance $\ell(t) = \int_0^t v(\tau)d\tau$



$$r(t) = \sqrt{d^2 + \ell^2(t) - 2d\ell(t)\cos\theta}$$

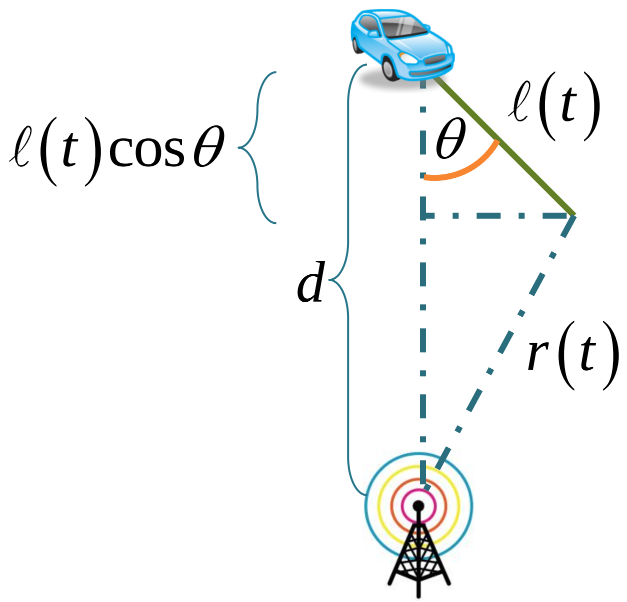
$$\frac{d}{dt}r(t) = \frac{2\ell(t) - 2d\cos\theta}{2\sqrt{d^2 + \ell^2(t) - 2d\ell(t)\cos\theta}}v(t)$$

$$\left.\frac{d}{dt}r(t)\right|_{t=0} = -\cos\theta v(0)$$

$$f_{\text{new}}(t) = f - \frac{1}{\lambda}\frac{d}{dt}r(t)$$

$$f_{\text{new}}(0) = f + \underbrace{\frac{1}{\lambda}\cos\theta v(0)}_{\text{Frequency shift}}$$

Doppler Shift: Approximation



$$r(t) \approx d - \ell(t)\cos\theta$$

$$\frac{d}{dt} r(t) \approx -v(t)\cos\theta$$

$$f_{\text{new}}(t) \approx f + \frac{v(t)\cos\theta}{\lambda}$$

$$\Delta f = \frac{v\cos\theta}{\lambda}$$

For typical vehicle speeds (75 Km/hr) and frequencies (around 1 GHz), it is on the order of 100 Hz 70 Hz

100 km/hr, 2GHz → 185 Hz.

[Goldsmith, 2005, Fig 2.2]

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation	✓Attenuation (Path Loss) ✓Shadowing ✓Doppler shift Inter-symbol interference (ISI) Flat fading Frequency-selective fading
Extraneous signals	Co-channel interference Adjacent channel interference Impulse noise White noise
Transmitting and receiving equipment	White noise Nonlinear distortion Frequency and phase offset Timing errors

[Myung and Goodman, 2008, Table 2.1]

ECS 455 Chapter 1

Introduction & Review

1.4 Spectrum Allocation

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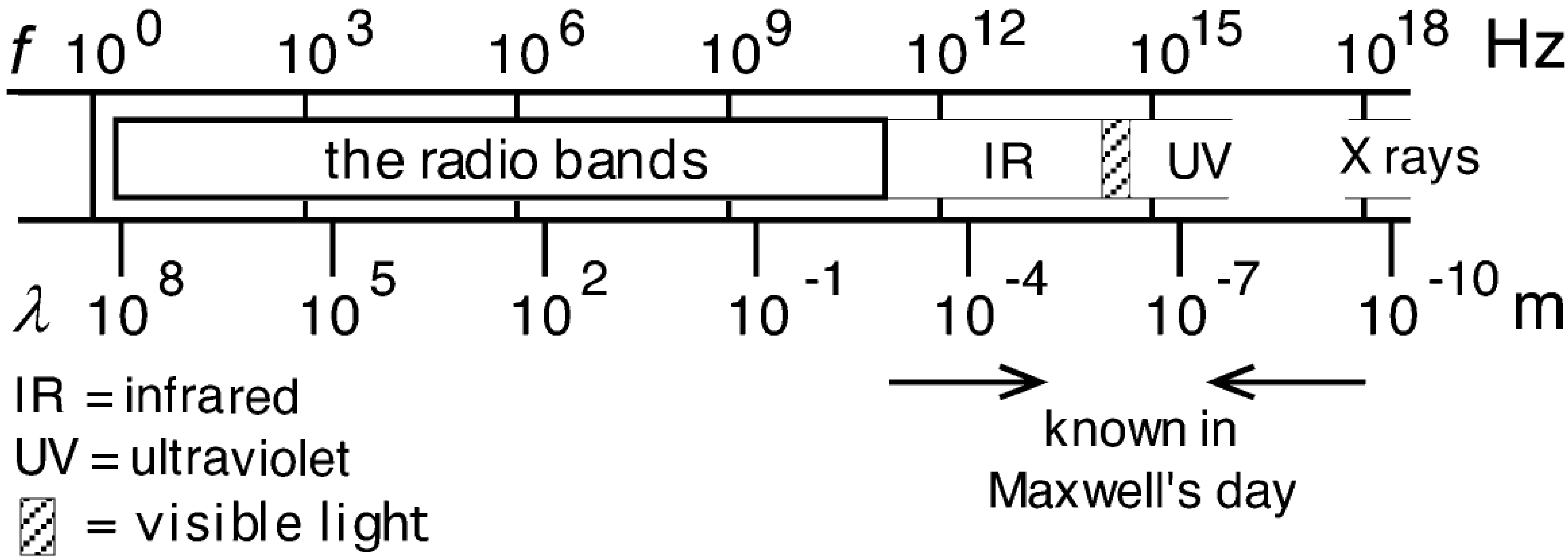
Friday 9:30-10:30

Electromagnetic Spectrum



[Gosling , 1999, Fig 1.1]

$$c = f\lambda$$

3×10^8 m/s

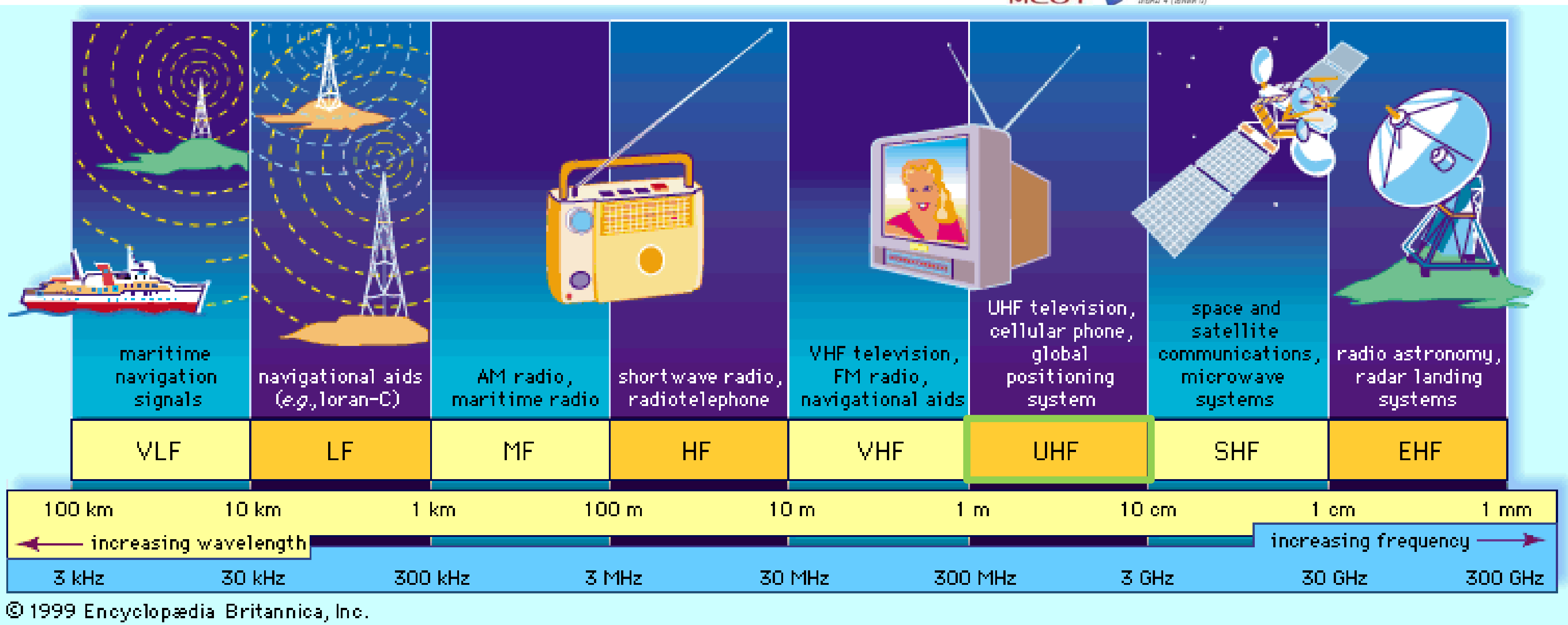
Wavelength

Frequency

[http://www.britannica.com/EBchecked/topic-art/585825/3697/Commercially-exploited-bands-of-the-radio-frequency-spectrum]

Radio-frequency spectrum

- Commercially exploited bands



$$c = f\lambda$$

3×10^8 m/s → c; Frequency → f; Wavelength → λ

Note that the freq. bands are given in decades; the VHF band has 10 times as much frequency space as the HF band.

Cellular Bands

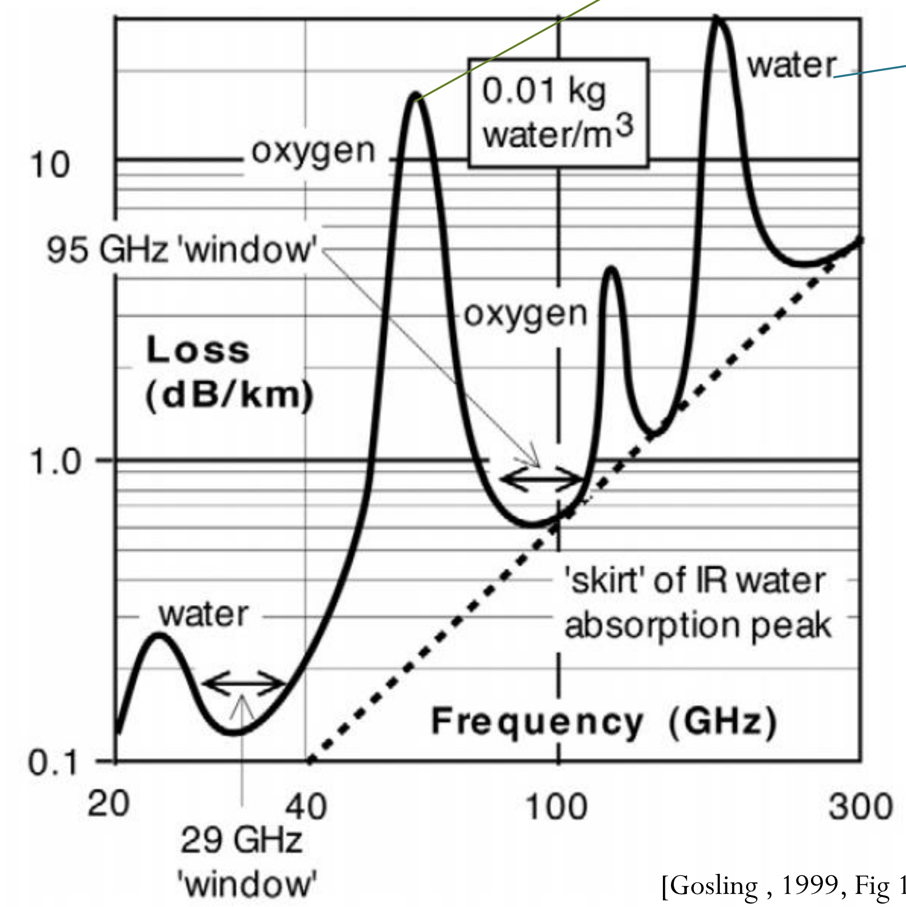
- All cellular phone networks worldwide use a portion of the radio frequency spectrum designated as **ultra high frequency (UHF)** (300 MHz to 3 GHz)
  - The UHF band is also used for television, Wi-Fi and Bluetooth transmission.
  - Due to historical reasons, radio frequencies used for cellular networks differ in the Americas, Europe, and Asia.
- Frequency bands recommended by ITU-R (in June 2003) for terrestrial Mobile telecommunication IMT-2000:
  - 806-960 MHz
  - 1710-2025 MHz
  - 2110-2200 MHz
  - 2500-2690 MHz

Lower limits on radio use

- **Efficiency** of an antenna in radiating radio energy is dependent on its length expressed as a fraction of **wavelength**.
  - Too low frequency = too large antenna
- Ex. The "Sanguine" submarine communication system
  - 30 Hz (10,000 km wavelength)
  - Designed (but never built) for the US Navy
  - Base antenna: 24 km square mesh of wires.
  - 10MW RF input
    - Radiate only 147W
    - All the remainder of the power dissipates as heat.

[Gosling, 1999, p 11]

Upper limits on radio use



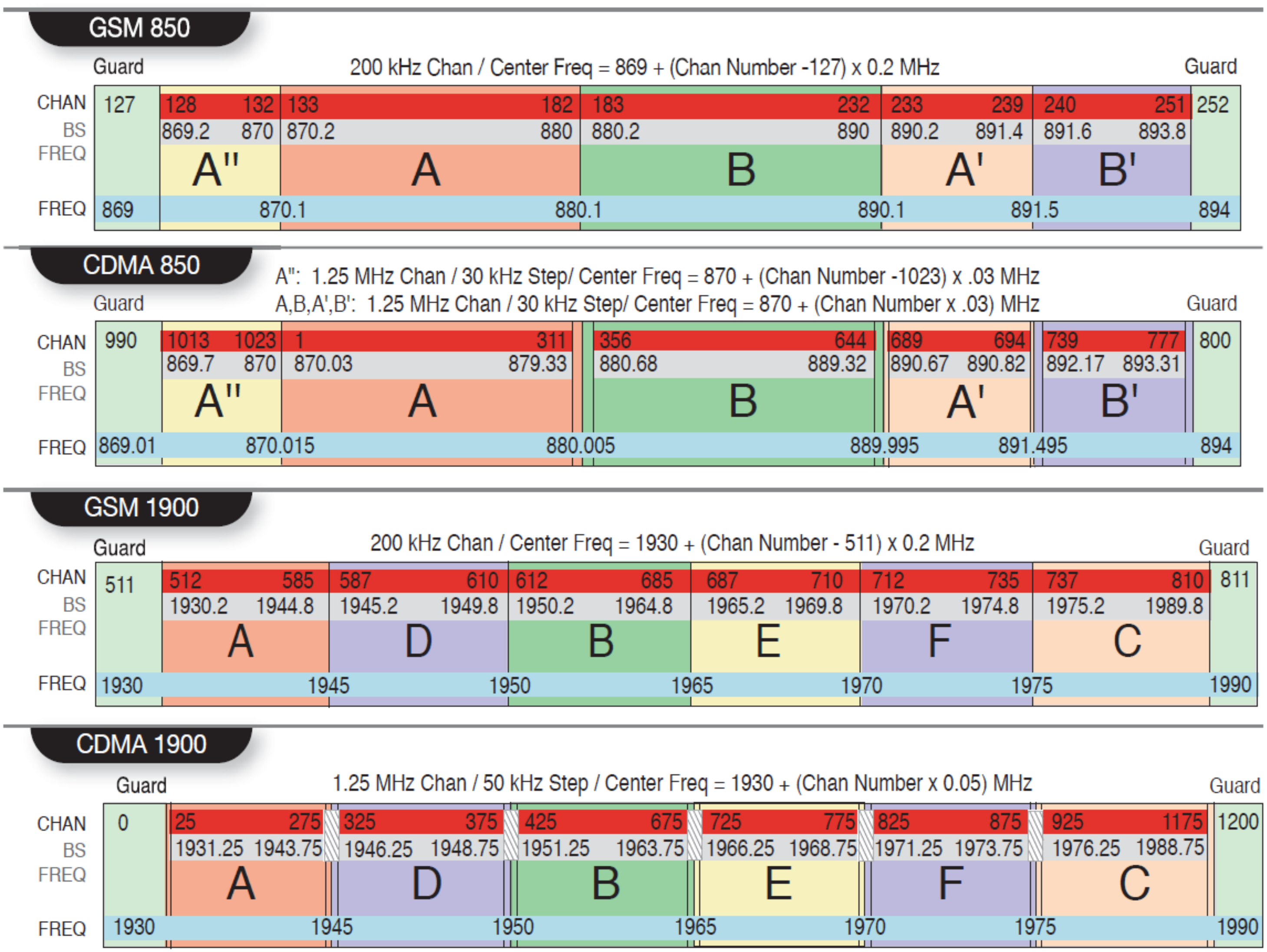
[Gosling , 1999, Fig 1.1]

Make commu. very dependent on weather conditions

- Atmospheric absorption
- Quasi-optical propagation
  - Short wavelength = Deep shadows behind obscuring objects = Unreliable coverage.
- Increased absorption by building and structural materials

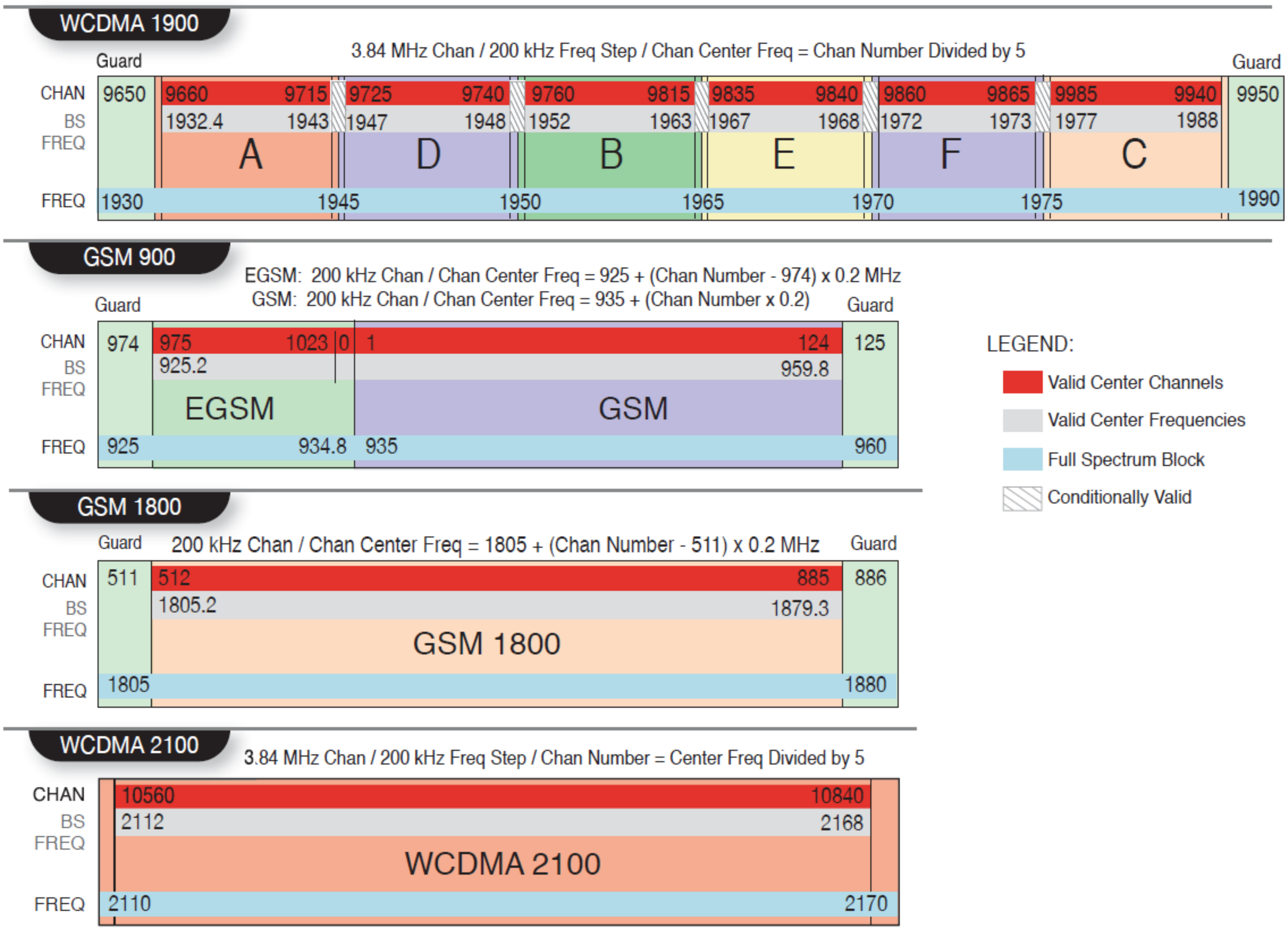
Forward link (BS to MS) Frequencies and Channelization (1)

[http://www.qrctech.com/freq_chart_24x36.pdf]



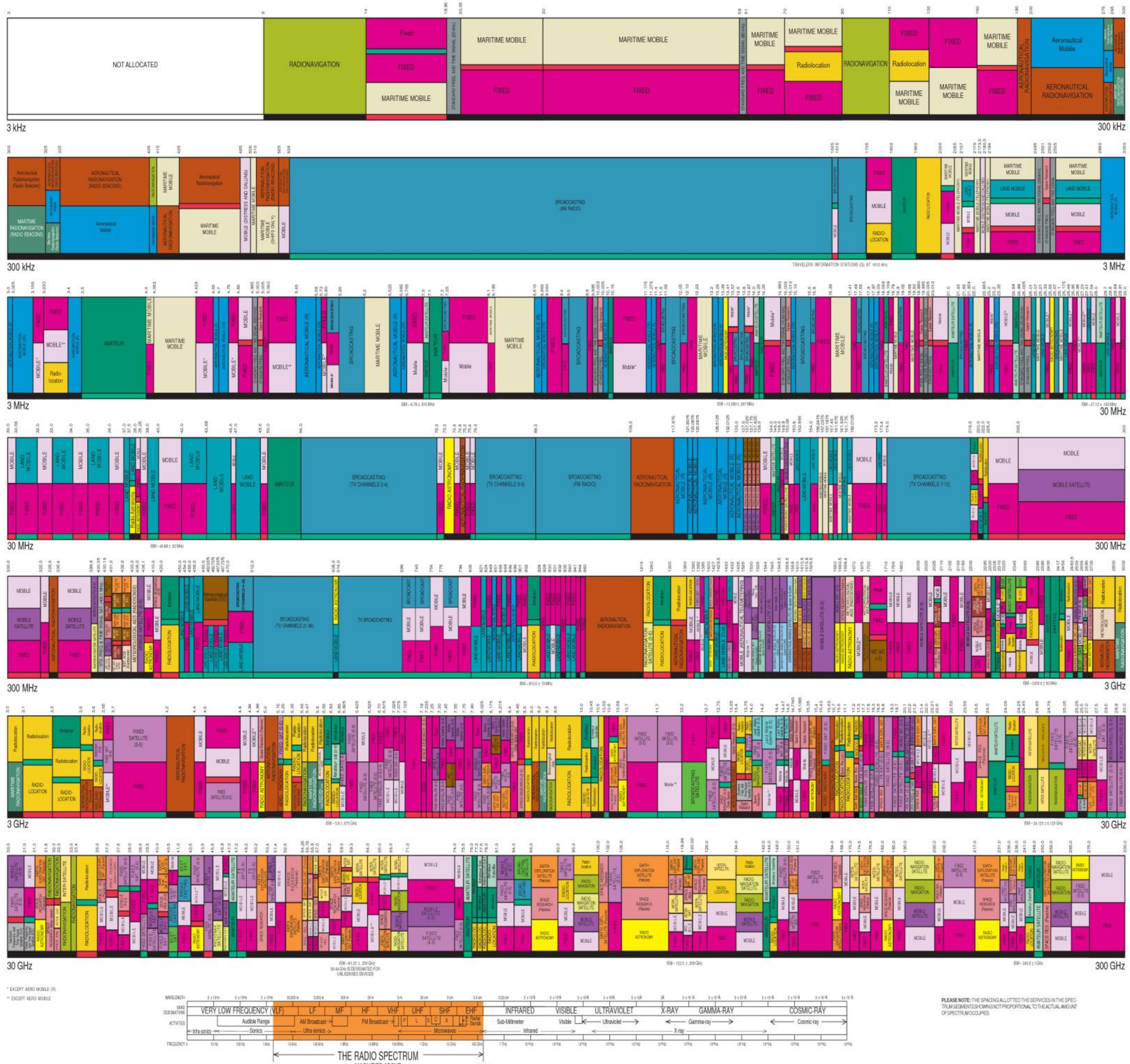
Forward link (BS to MS) Frequencies and Channelization (2)

[http://www.qrctech.com/freq_chart_24x36.pdf]



UNITED STATES FREQUENCY ALLOCATIONS

THE RADIO SPECTRUM



RADIO SERVICES COLOR LEGEND

- AERONAUTICAL MOBILE
- AERONAUTICAL MOBILE SATELLITE
- AERONAUTICAL RADIONAVIGATION
- AMATEUR
- AMATEUR SATELLITE
- BROADCASTING
- BROADCASTING SATELLITE
- EARTH EXPLORATION SATELLITE
- FIXED
- FIXED SATELLITE
- INTER-SATELLITE
- LAND MOBILE
- LAND MOBILE SATELLITE
- MARITIME MOBILE
- MARITIME MOBILE SATELLITE
- MARITIME RADIONAVIGATION
- METEOROLOGICAL AIDS
- METEOROLOGICAL SATELLITE
- MOBILE
- MOBILE SATELLITE
- RADIO ASTRONOMY
- RADIODETERMINATION SATELLITE
- RADIOLOCATION
- RADIOLOCATION SATELLITE
- RADIONAVIGATION
- RADIONAVIGATION SATELLITE
- SPACE OPERATION
- SPACE RESEARCH
- STANDARD FREQUENCY AND TIME SIGNAL
- STANDARD FREQUENCY AND TIME SIGNAL SATELLITE

ACTIVITY CODE

- GOVERNMENT EXCLUSIVE
- GOVERNMENT/NON-GOVERNMENT SHARED
- NON-GOVERNMENT EXCLUSIVE

ALLOCATION USAGE DESIGNATION

SERVICE	EXAMPLE	DESCRIPTION
Primary	FIXED	Capital Letters
Secondary	Mobile	1st Capital with lower case letters

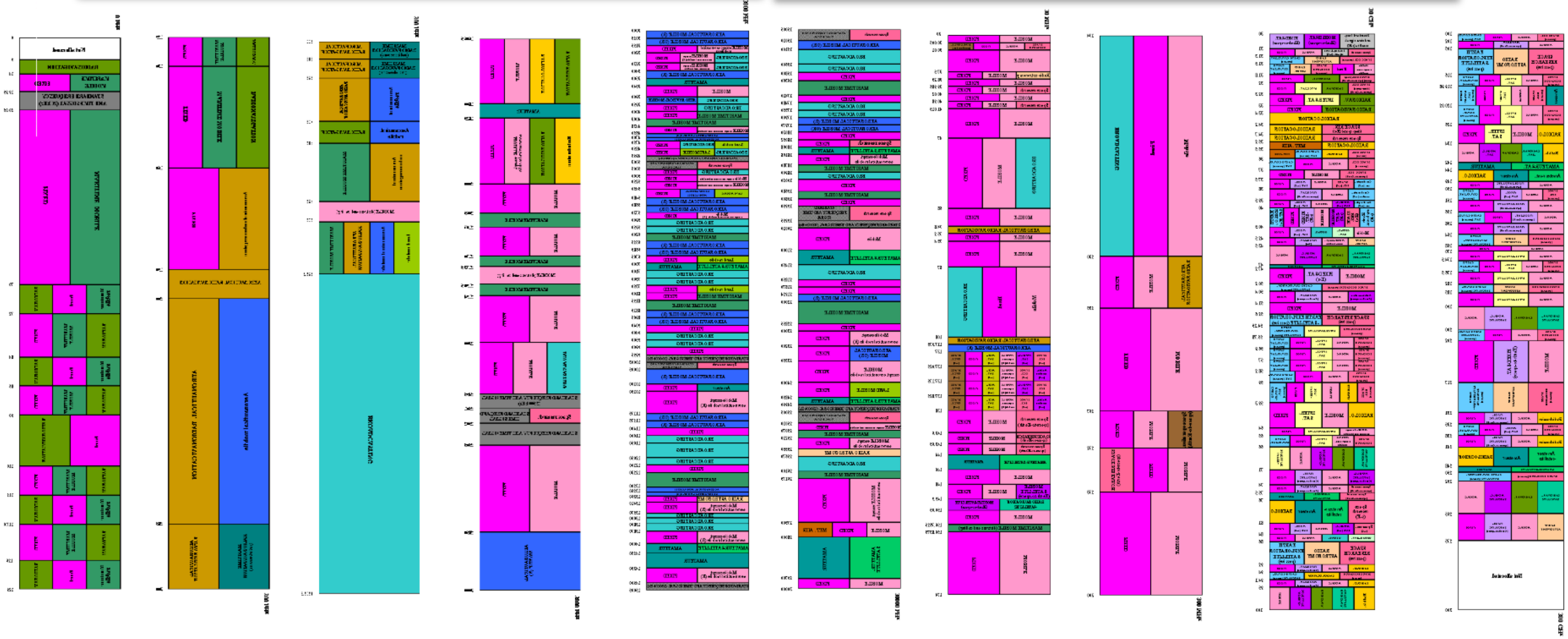
This chart is a graphic single-point-in-time portrayal of the Table of Frequency Allocations used by the ... NTIA. As such, it does not completely reflect all aspects, i.e., footnotes and recent changes ... Table of Frequency Allocations. Therefore, for complete information, users should consult the ... the current status of U.S. allocations.

U.S. DEPARTMENT OF COMMERCE
National Telecommunications and Information Administration
Office of Spectrum Management
October 2003

Thailand Freq. Allocations Chart

RADIO SERVICES COLOR LEGEND	
Aeronautical mobile	Meteorological aids
Aeronautical radionavigation	Meteorological-satellite
Amateur	Mobile
Amateur-satellite	Mobile-satellite
Broadcasting	Radio astronomy
Broadcasting-satellite	Radiodetermination-satellite
Earth exploration- satellite	Radiolocation

Fixed	Radionavigation
Fixed-satellite	Radionavigation- satellite
Inter-satellite	Space operation
Land mobile	Space research
Maritime mobile	Standard frequency and time signal
Maritime radionavigation	Standard frequency and time signal-satellite



http://www.ntc.or.th/uploadfiles/freq_chart_thai.htm

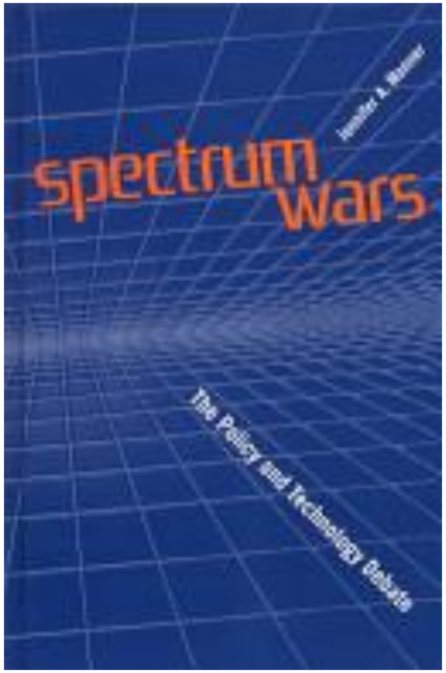
Spectrum Allocation

- Spectral resource is limited.
- Most countries have government agencies responsible for allocating and controlling the use of the radio spectrum.
- Commercial spectral allocation is governed
  - **globally** by the International Telecommunications Union (**ITU**)
    - ITU Radiocommunication Sector (**ITU-R**) is responsible for radio communication.
  - in the U.S. by the Federal Communications Commission (**FCC**)
  - in Europe by the European Telecommunications Standards Institute (ETSI)
  - in Thailand by the National Telecommunications Commission (**NTC**; คณะกรรมการกิจการโทรคมนาคมแห่งชาติ; กทช.)
    - replaced by the National Broadcasting and Telecommunications Commission (**NBTC**; คณะกรรมการกิจการกระจายเสียง กิจการโทรทัศน์และกิจการโทรคมนาคมแห่งชาติ ; กสทช.)
- Blocks of spectrum are now commonly assigned through **spectral auctions** to the highest bidder.

Interesting Book

- Spectrum Wars: The Policy and Technology Debate

"Designed to help you ensure that your company **wins the battle for the spectrum**, this text maps out the strategies required for structuring entry and operations in the spectrum. It offers advice on how to master the lobbying, technical, regulatory, legal and political tools needed for success."



[Manner, 2003]

US licensed spectrum

AM Radio	535-1605 KHz
FM Radio	88-108 MHz
Broadcast TV (Channels 2-6)	54-88 MHz
Broadcast TV (Channels 7-13)	174-216 MHz
Broadcast TV (UHF)	470-806 MHz
3G Broadband Wireless	746-764 MHz, 776-794 MHz
3G Broadband Wireless	1.7-1.85 MHz, 2.5-2.69 MHz
1G and 2G Digital Cellular Phones	806-902 MHz
Personal Communications Service (2G Cell Phones)	1.85-1.99 GHz
Wireless Communications Service	2.305-2.32 GHz, 2.345-2.36 GHz
Satellite Digital Radio	2.32-2.325 GHz
Multichannel Multipoint Distribution Service (MMDS)	2.15-2.68 GHz
Digital Broadcast Satellite (Satellite TV)	12.2-12.7 GHz
Local Multipoint Distribution Service (LMDS)	27.5-29.5 GHz, 31-31.3 GHz
Fixed Wireless Services	38.6-40 GHz

Unlicensed bands

- Frequency bands that are **free to use**
  - according to a specific set of **etiquette rules**.
- The purpose of these unlicensed bands is to encourage innovation and low-cost implementation.
- Many extremely successful wireless systems operate in unlicensed bands, including **wireless LANs, Bluetooth, and cordless phones**.
- Major difficulty:
  - If many unlicensed devices in the same band are used in close proximity, they generate much **interference** to each other, which can make the band unusable.

Unlicensed bands (2)

- Unlicensed spectrum is allocated by the governing body within a given country.
- Often countries try to match their frequency allocation for unlicensed use so that technology developed for that spectrum is compatible worldwide.
- The following table shows the unlicensed spectrum allocations in the U.S.

(ISM = Industrial, Scientific, and Medical)

900 MHz	ISM Band I (Cordless phones, 1G WLANs)	902-928 MHz
2.4 GHz	ISM Band II (Bluetooth, 802.11b WLANs)	2.4-2.4835 GHz
5.8 GHz	ISM Band III (Wireless PBX)	5.725-5.85 GHz
5 GHz	NII Band I (Indoor systems, 802.11a WLANs)	5.15-5.25 GHz
5 GHz	NII Band II (short outdoor and campus applications)	5.25-5.35 GHz
5.8 GHz	NII Band III (long outdoor and point-to-point links)	5.725-5.825 GHz

(U-NII = Unlicensed National Information Infrastructure)

[Tse Viswanath, 2005, Section 4.1]

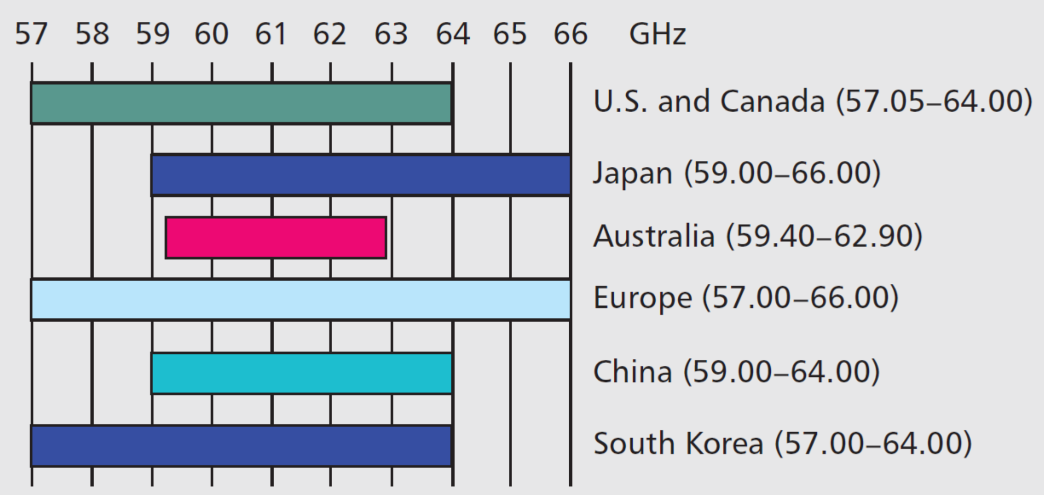
Licensed vs. Unlicensed Spectra

Licensed	Unlicensed
Typically nationwide. Over a period of a few years. From the spectrum regulatory agency.	For experimental systems and to aid development of new wireless technologies.
Bandwidth is very expensive.	Very cheap to transmit on.
No hard constraints on the power transmitted within the licensed spectrum but the power is expected to decay rapidly outside.	There is a maximum power constraint over the entire spectrum.
Provide immunity from any kind of interference outside of the system itself.	Have to deal with interference.

Unlicensed 60 GHz Frequency Band

- A lot of bandwidth available

Worldwide spectrum availability

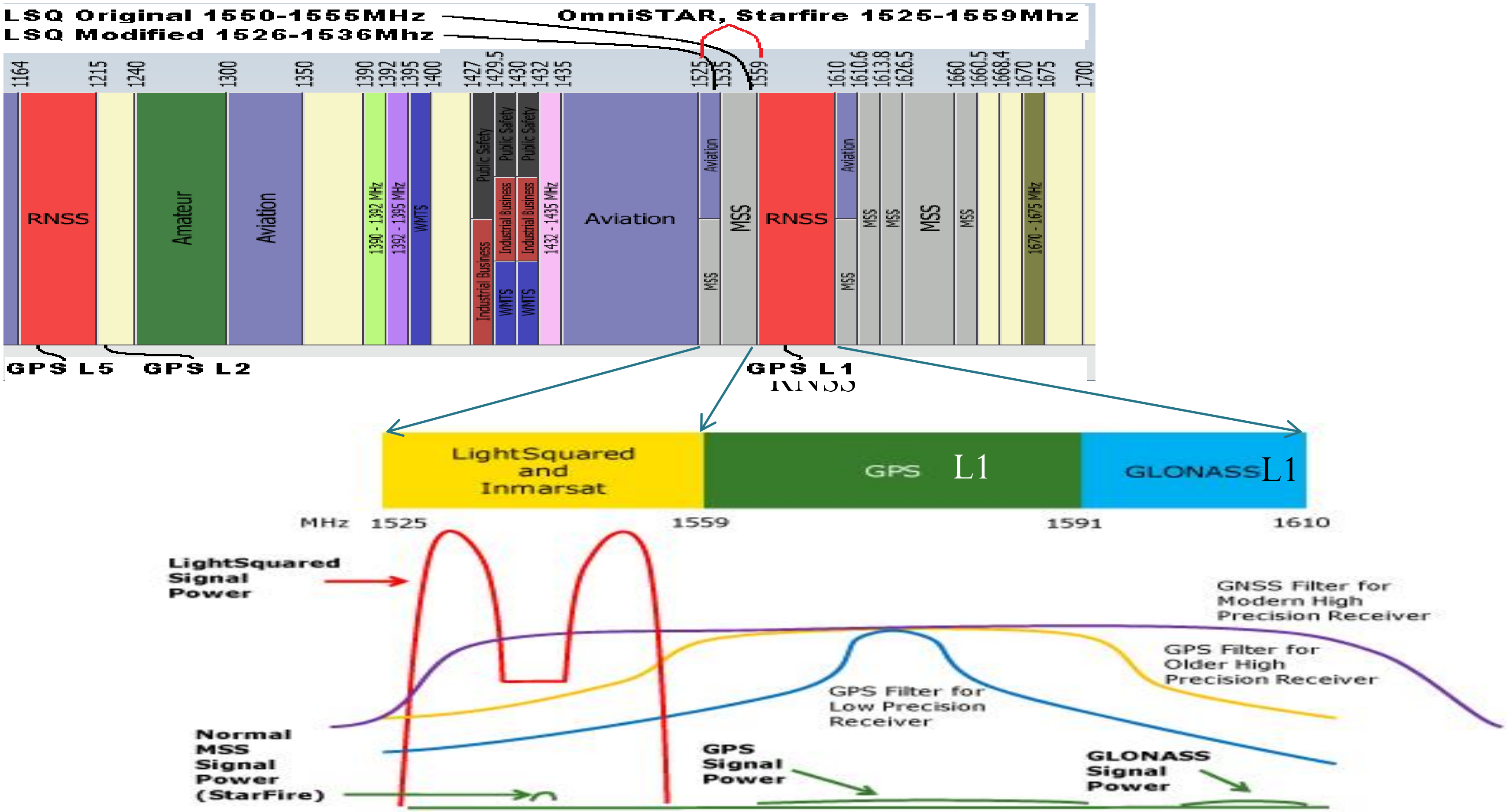


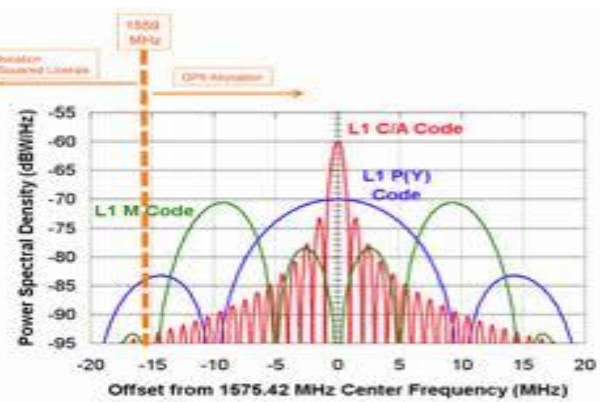
- Even for the smallest allocation, there is more than 3 GHz of bandwidth available, and most regions allow use of at least **7 GHz**.
  - In comparison, the 5 GHz unlicensed band has about 500 MHz of total usable bandwidth.
  - The 2.4 GHz band has less than 85 MHz of bandwidth in most regions.

News: LightSquared vs. GPS industry

- The FCC recently (Jan 2011) granted a conditional waiver to **LightSquared** allowing the expansion of terrestrial use (for launching a new **LTE** network) of the **mobile satellite spectrum (MSS)** immediately neighboring that of the **GPS**
  - As its name suggested, MSS has been reserved for satellite services
  - Earlier, FCC permitted "ancillary" terrestrial uses intended to "fill in" locations where satellite coverage was problematic.
  - The new order allows a high powered nationwide terrestrial broadband network.
- Extremely high-powered ground-based transmissions could potentially cause severe interference to GPS receivers.
- LightSquared bought the spectrum right next door to GPS cheaply, hoping to change the rules and make the spectrum more valuable.

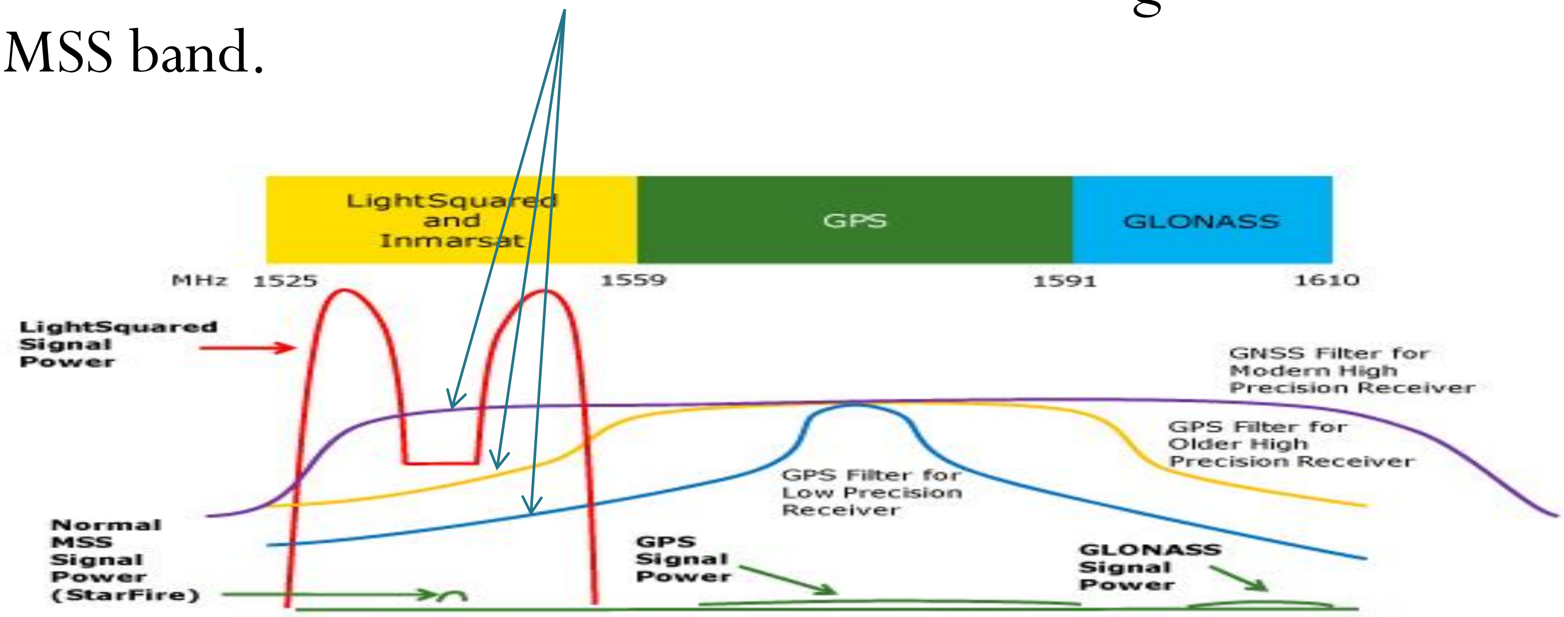
[GPS World, December 2011]


LSQ Original 1550-1555MHz
LSQ Modified 1526-1536Mhz
OmniSTAR, Starfire 1525-1559Mhz
1164
1215
1240
1300
1350
1390
1392
1395
1400
1427
1429.5
1430
1432
1435
1525
1535
1559
1610
1610.6
1613.8
1626.5
1660
1660.5
1668.4
1670
1675
1700
RNSS
Amateur
Aviation
1390 - 1392 MHz
1392 - 1395 MHz
WMTS
Public Safety
Industrial Business
1432 - 1435 MHz
Aviation
MSS
RNSS
1670 - 1675 MHz
GPS L5
GPS L2
GPS L1
RNSS
LightSquared and Inmarsat
GPS L1
GLONASS L1
MHz 1525
1559
1591
1610
LightSquared Signal Power
GNSS Filter for Modern High Precision Receiver
GPS Filter for Older High Precision Receiver
GPS Filter for Low Precision Receiver
Normal MSS Signal Power (StarFire)
GPS Signal Power
GLONASS Signal Power


1559 MHz
L1 C/A Code
L1 P(Y) Code
L1 M Code
Power Spectral Density (dBW/Hz)
-55
-60
-65
-70
-75
-80
-85
-90
-95
-20
-15
-10
-5
0
5
10
15
20
Offset from 1575.42 MHz Center Frequency (MHz)

Completely Separated?

- GPS receivers have filters that do not block signals from the MSS band.



- These filters has enabled both low-cost and high-precision GPS receivers.
- Assumption: Signals in MSS band were low-power.

http://www.saveourgps.org/

Spectrum Allocation (Final Words)

- Spectrum is a scarce resource.
- Spectrum is allocated in "chunks" in **frequency** domain.
  - "Chunks" are licensed to (cellular/wireless) operators.
- Within a single cellular operator, the chunk is further divided into many **channels**.
  - Each channel has its own band of frequency.
- Mobile networks based on different standards may use the same "frequency chunk".
  - For example, AMPS, D-AMPS, N-AMPS and IS-95 all use the 800 MHz "frequency chunk".
  - This is achieved by the use of **different channels**.

ECS 455 Chapter 2

Cellular Systems

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Pre-Cellular System

Area over which **reliable** radio comunication can occur btw a BS and MSs.

- Achieve a **large coverage area** by using a single, **high powered** transmitter.
  - Put BS on top of mountains or tall towers
- Next BS was so **far away** that interference was not an issue.
- Severely limit the number of users that could communicate simultaneously.
- Noise-limited system with few users.
- Bell mobile system in New York City in the 1970s could only support a maximum of twelve simultaneous calls over a thousand square miles.

Examples

- Using a typical analog system, each channel needs to have a bandwidth of around **25 kHz** to enable sufficient audio quality to be carried, as well as allowing for a guard band between adjacent signals to ensure there are no undue levels of interference.
- Can accommodate only **40 users** in a frequency "chunk" of 1-MHz wide.
- Even if **100 MHz** were allocated to the system, this would enable only **4000 users** to have access to the system.
- Today cellular systems have millions of subscribers, and therefore a far more efficient method of using the available spectrum is needed.

Pre-Cellular System (2)

- All regions use the **same group of frequencies**.
- Non-overlapping coverage of the regions is NOT enough

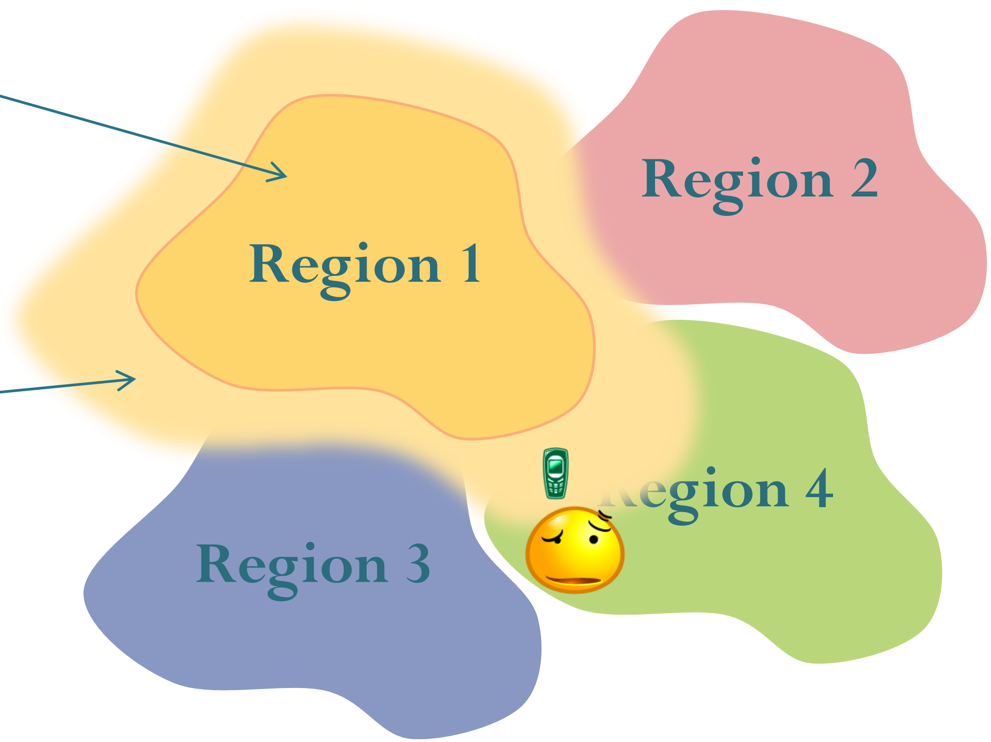
Colored area indicates the coverage area of the central transmitter in the corresponding region

Pre-Cellular System (2)

- All regions use the same group of frequencies.
- Non-overlapping coverage of the regions is NOT enough

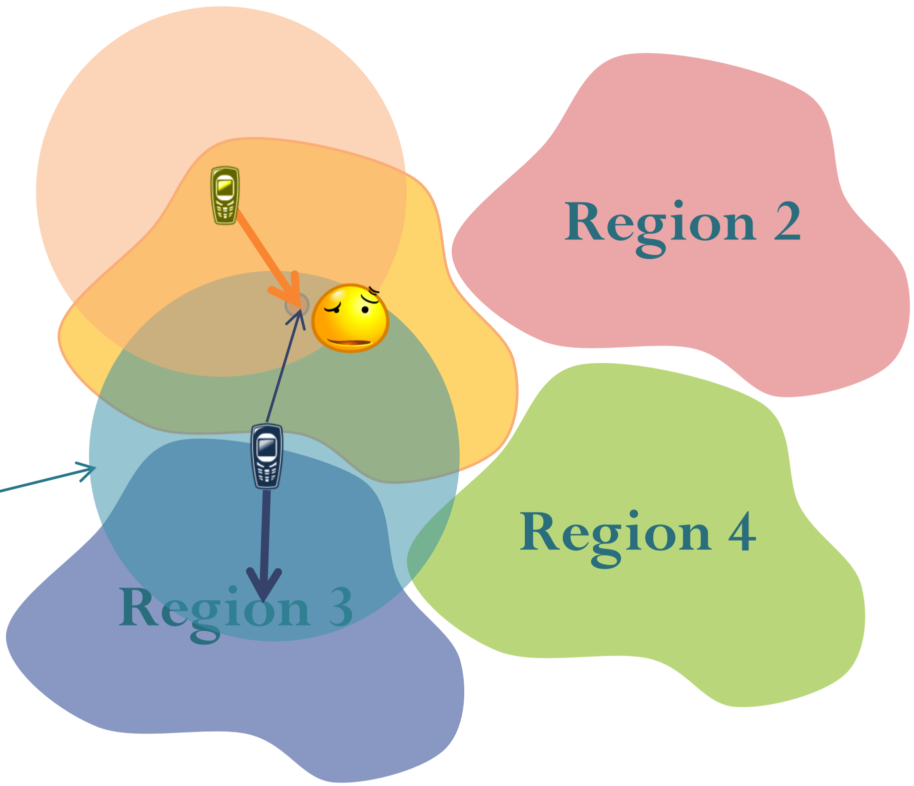
Downlink signal in this region is strong enough for communication

The signal in this region is ***not*** strong enough for communication but still impose significant ***interference*** on users in other regions



Pre-Cellular System (2)

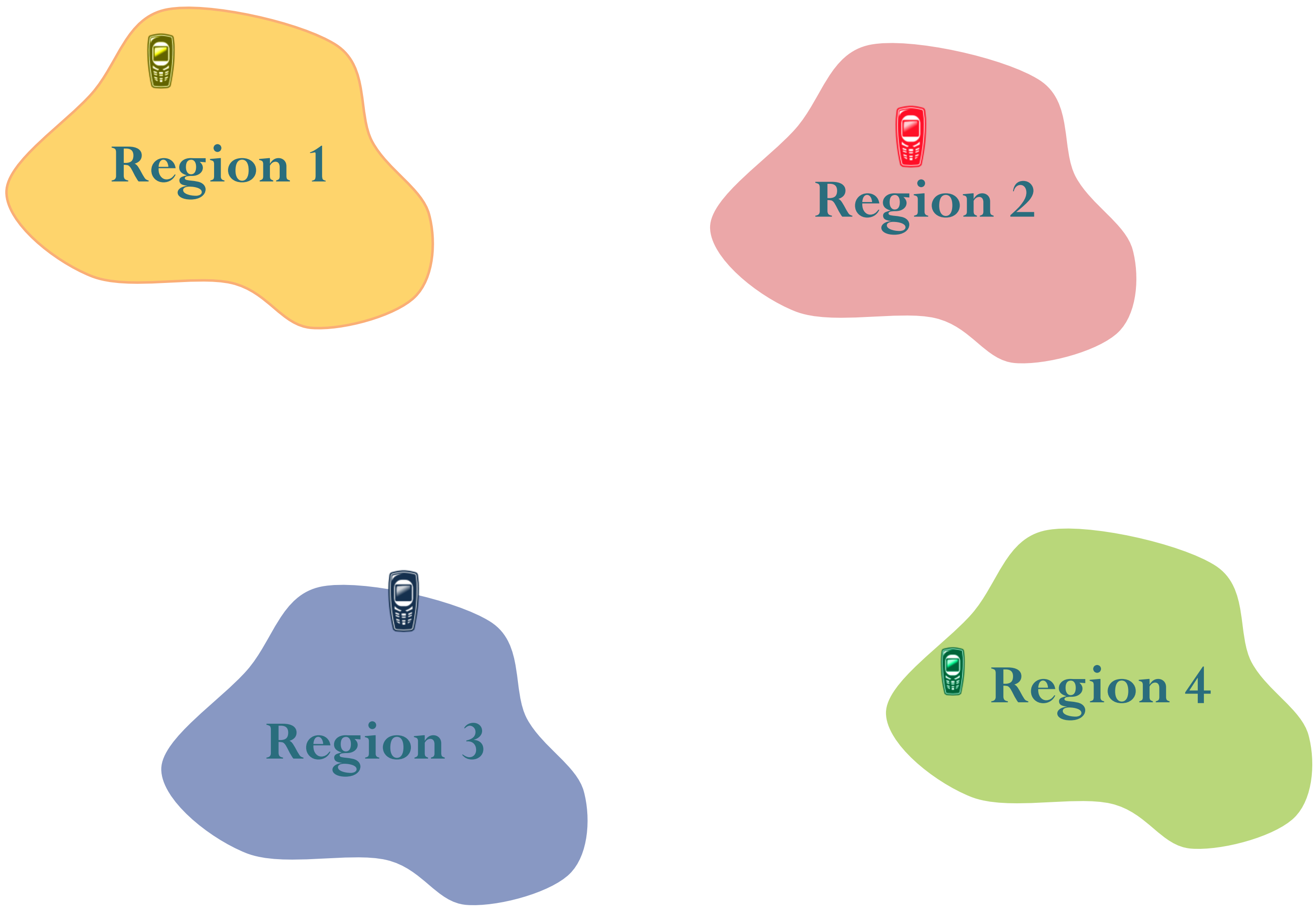
- All regions use the same group of frequencies.
- Non-overlapping coverage of the regions is NOT enough



Uplink signals from user of a cell can reach the BS of a different region.

Pre-Cellular System (3)

- Regions need to be well-separated!



ECS455 Chapter 2

Cellular Systems

2.1 Frequency Reuse

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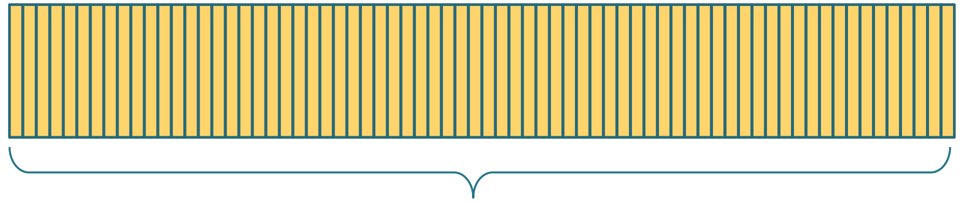
Cellular systems

- The coverage area is divided into many small areas (**cells**).
- Replace
  - a single, high power transmitter

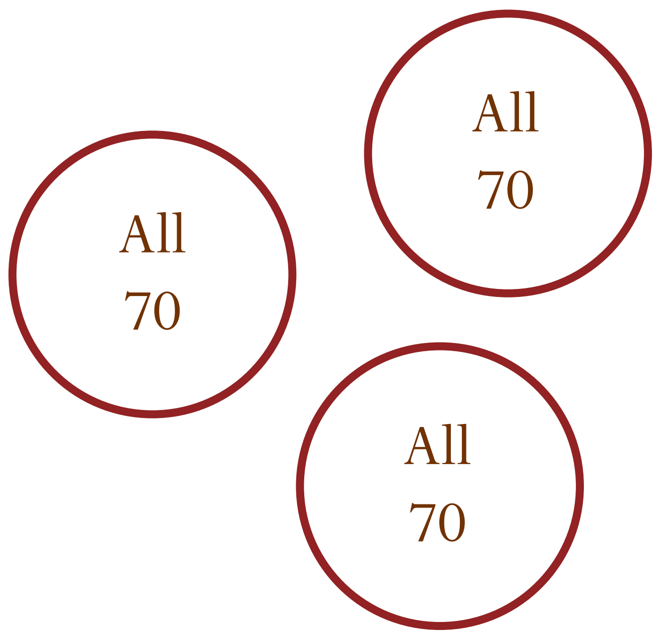
  with
  - **many low-power** transmitters each providing **coverage** to only one cell area (a small portion of the service area).
    - Power is lowered from hundreds of watts to a few watts, or even less than one watt per channel. [Klemens, 2010]

  → Area over which **reliable** radio comunication can occur btw a BS and MSs.
- **Frequency**/channel **Reuse**: Divide the available channels (frequency bands) into groups/sets. Different channel sets are assigned to different cells. The same channel sets may be **reused** at **spatially separated** locations.
- **Co-channel** cells = Cells that are assigned the same channel set

Idea (1)

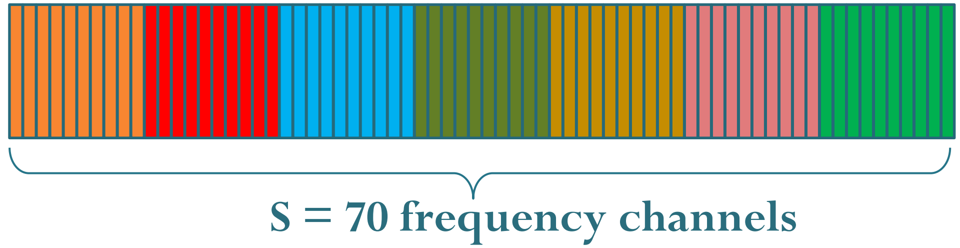


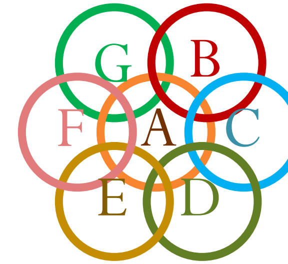
- Suppose the whole system has **S = 70 frequency channels**
- Pre-cellular:



"Capacity" of the system
= # users the system can support simultaneously
= $70 \times 3 = 210$

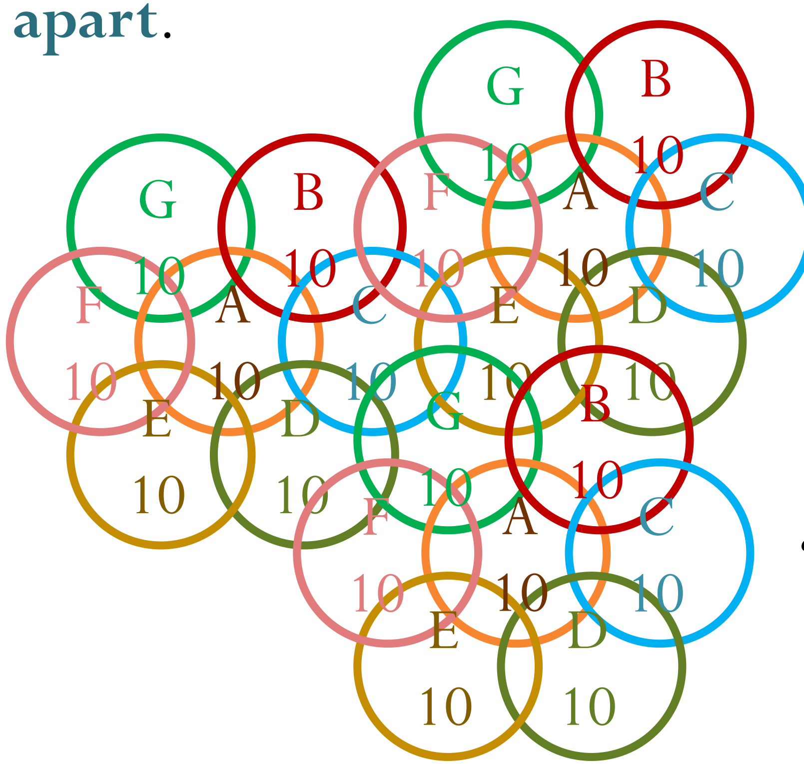
Idea (2)





- Cellular:
  - Split 70 channels into 7 groups (A,B,C,D,E,F,G).
  - Each group has m = 10 channels. Cells using the same groups are **far apart**.

Less interference (Recall that P_r is inversely proportional to d^{γ}.)



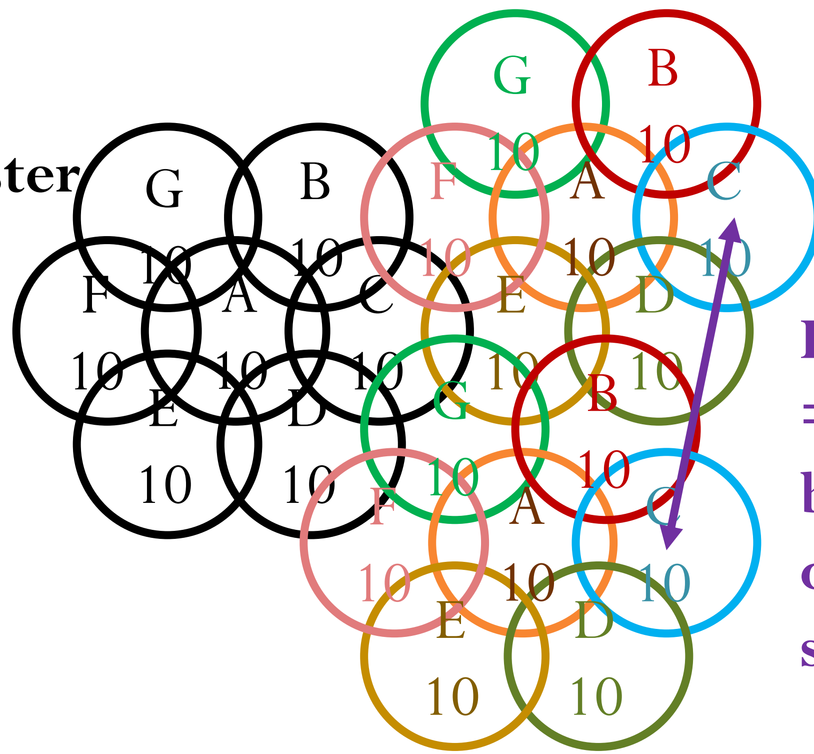
Note:
Cells can overlap.

"Capacity" of the system
= # users the system can support simultaneously
= 70×3 = 210

Idea (3)

- Some Terminology:

Cluster

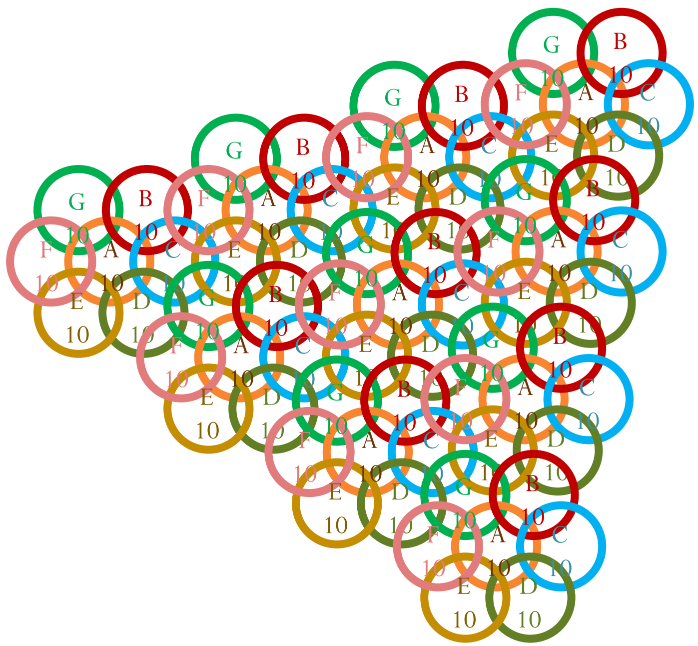


Cluster size
N = 7

Reuse Distance (D) = minimum distance between the centers of cells that use the same channels

Idea (4)

- To support more users (increase capacity), simply use smaller cell size (area).



“Capacity” of the system
= # users the system can
support simultaneously
>> 210

Cellular systems: Handoff

- Sophisticated **switching** technique
- Enable a call to proceed **uninterrupted** when the user moves from **one cell to another**.
- The system can switch moving users between towers to find the **strongest signal**.

Can we keep reducing the cell size?

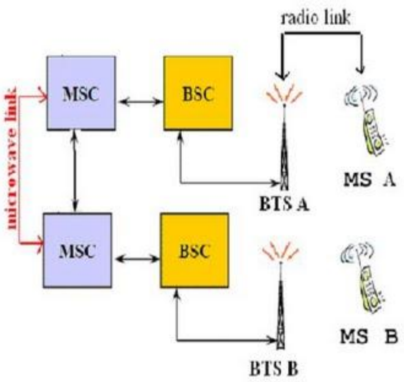
- While smaller cells generally increase capacity, they also have their disadvantages.
- Smaller cell size increases the rate at which **handoffs** occur, which increases the dropping probability if the percentage of failed handoffs stays the same.
- Smaller cells increase the **load** on the backbone network.
- More cells per unit area requires more base stations, which can increase system **cost**.
- **Propagation** characteristics typically change as cell size shrinks, so the system does not scale perfectly.

[Goldsmith, 2005, p. 471]

Cellular systems: History

- The concept of cells was first proposed (in an unpublished work) as early as **1947** by Douglas H. Ring at **Bell Lab**oratories in the US
- Detailed proposal for a "High-Capacity Mobile Telephone System" incorporating the cellular concept submitted by Bell Laboratories to the FCC in 1971.
- The first **AMPS** system was deployed in Chicago in **1983**.

Basic cellular system



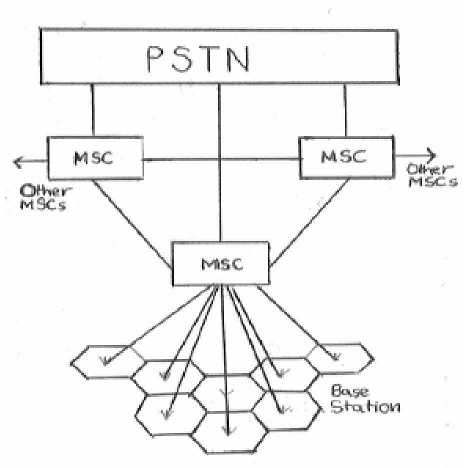
1. **Mobile stations (MS)**
2. **Base stations (BS)**
   - Serve as a bridge between all mobile users in the cell and connects the simultaneous mobile calls via telephone lines or microwave links to the MSC.
   - Generally have towers which support several transmitting and receiving antennas.
     - Simultaneously handle full duplex communications.

- Each mobile communicates via radio with one of the base stations and may be handed-off to any number of base stations throughout the duration of a call.

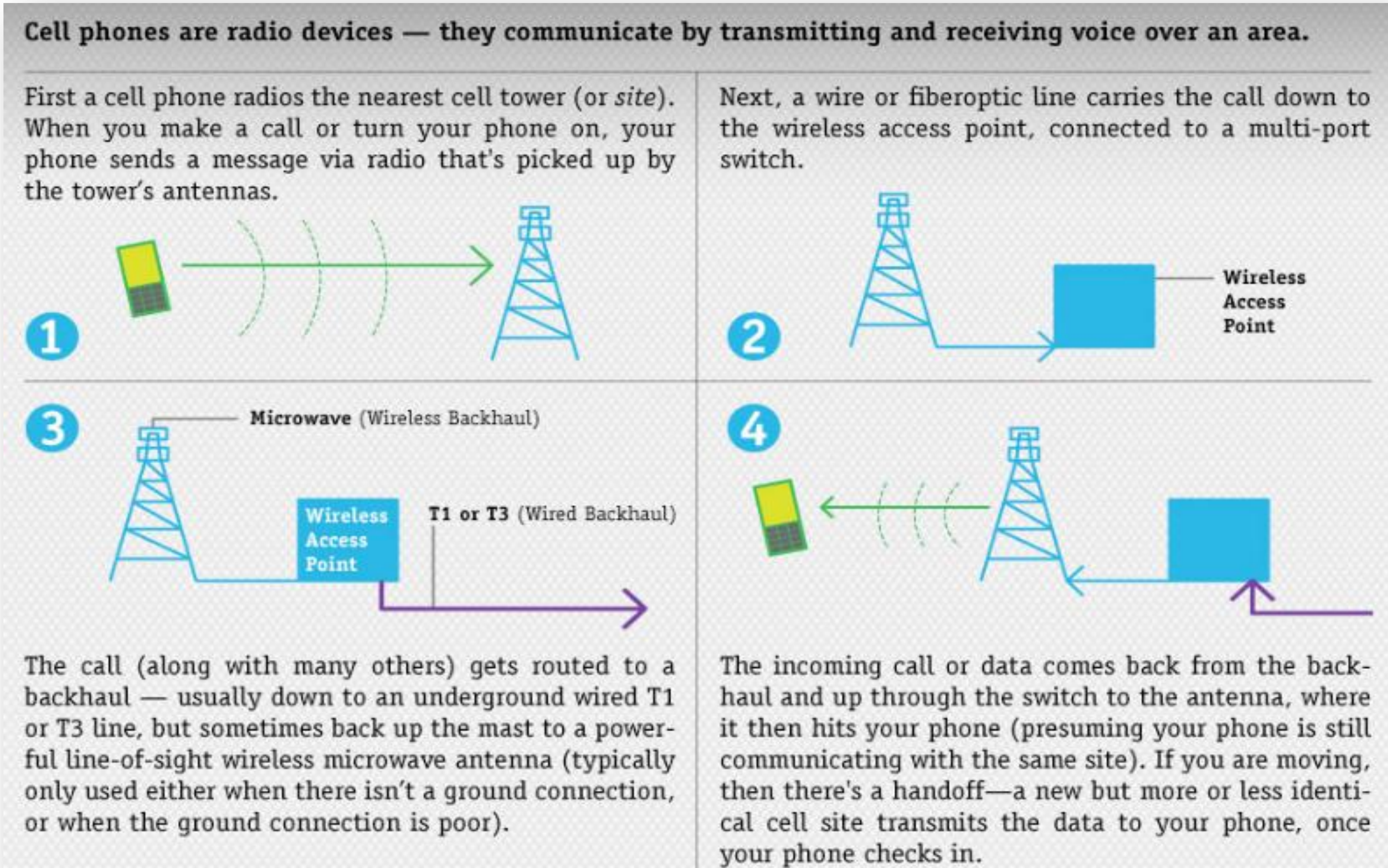
Basic cellular system (2)

3. **Mobile switching center** (**MSC**)

- Sometimes called a **mobile telephone switching office (MTSO)**
- **Coordinates** the activities of all of the base stations
  - Coordinating which BS will **handle** a call to or from a user and when to **handoff** a user from one base-station to another.
- **Connect** the entire cellular system to the **PSTN** (public switched telephone network).



How a Cell Phone Call Works



[http://cellphones.org/blog/how-cell-phone-calls-work/]

Common Air Interface (CAI)

- Standard for communication between BS and MSs

1. **Voice channels**
   - **Forward voice channels** (**FVC**) : voice transmission from BS to MSs
   - **Reverse voice channels** (**RVC**): voice transmission from MSs to BS
2. **Control channels**
   - Often called **setup channels**
   - **Forward control channels** (**FCC**) and **reverse control channels** (**RCC**)
   - Involve in setting up a call and moving it to an unused voice channel.
   - Transmit and receive data messages that carry call initiation and service requests
   - Monitored by mobiles when they do not have a call in progress.

- Typically, 5% control channels and 95% voice channels.

Frequency Reuse (Review)

Definition

"The use of radio channels on the **same carrier frequency** to cover **different areas** which are separated from one another by sufficient distances so that **co-channel interference** is not objectionable."

[Mac Donald, 1979, p 16]

- Employed not only in mobile-telephone service but also in entertainment broadcasting and many other radio services.

Frequency Reuse (Review)

- Cellular radio systems rely on an intelligent allocation and reuse of channels throughout a coverage region
- Each cellular BS is allocated a **group** of radio channels to be used within the corresponding cell.
- BSs in **adjacent** cells are assigned channel groups which contain completely **different** channels than neighboring cells.
- By limiting the coverage area to within the boundaries of a cell, the same group of channel may be used to cover different cells that are separated from one another by distances large enough to keep interference levels within tolerable limits.
- The distance between two cells that use the same frequency channels is called the **reuse distance**.

[Rappaport, 2002]

Cell Shape

- The actual radio coverage of a cell is known as the **footprint**.
  - Determined from field measurements or propagation prediction models.
- In reality, it is **not possible to define exactly the edge of a cell**.
  - Signal strength gradually reduces, and towards the edge of the cell performance falls.
  - MSs have different levels of sensitivity, this adds a further greying of the edge of the cell.
  - Impossible to have a sharp cut-off between cells.
- In some areas they may overlap, whereas in others there will be a hole in coverage.
- Although the real footprint is amorphous in nature, a **regular** cell shape is needed for systematic system design and adaptation for future growth.

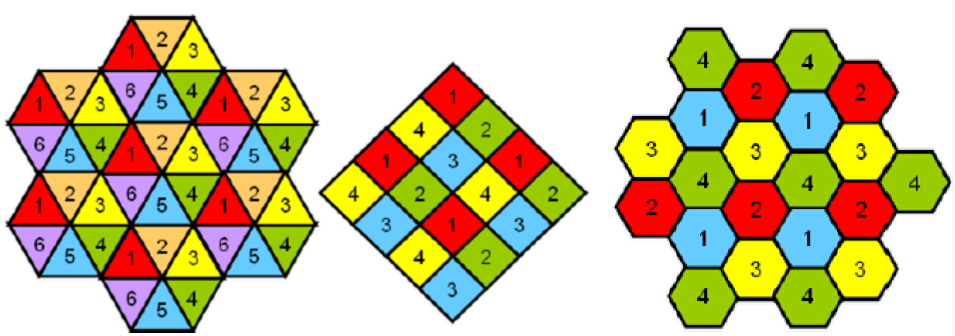
[Rappaport, 2002]

Hexagonal cell shape

- Simplistic model of the radio coverage for each BS.
- Universally adopted
- Permit easy and manageable analysis

Why hexagon?

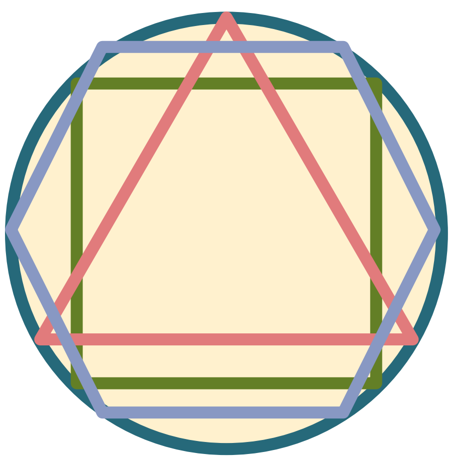
- Omnidirectional BS antenna and free space propagation → Circular radiation pattern.
  - Adjacent **circles** cannot be overlaid upon a map without leaving gaps or creating overlapping regions.
- **Tessellating Cell Shapes**: When considering geometric shapes which **cover** an entire region **without overlap** and with equal area, there are three sensible choices: a square, an equilateral triangle, and a hexagon.



Diamond and rectangles are also tessellating shapes.

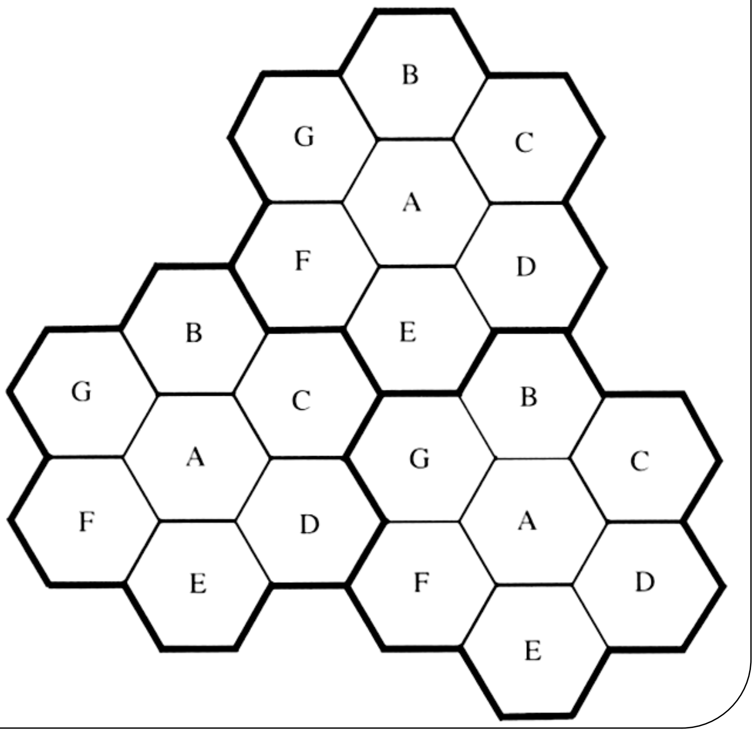
Why hexagon? (2)

- A cell must be designed to **serve** the **weakest** mobiles within the footprint, and these are typically located at the **edge** of the cell.
  - For a given distance between the center of a polygon and its farthest perimeter points, the hexagon has the **largest area** of the three.
  - By using the hexagon geometry, the **fewest** number of cells can cover a geographic region



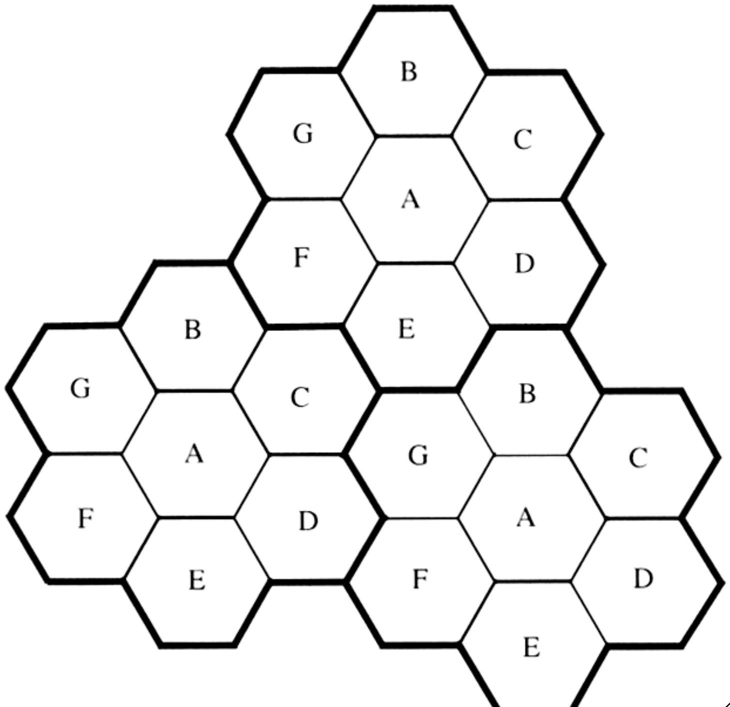
Frequency Reuse Plan

- The **frequency reuse plan** is overlaid upon a map to indicate where different frequency channels are used.
- Cells labeled with the same letter use the same group of channels.
  - Create **co-channel interference**

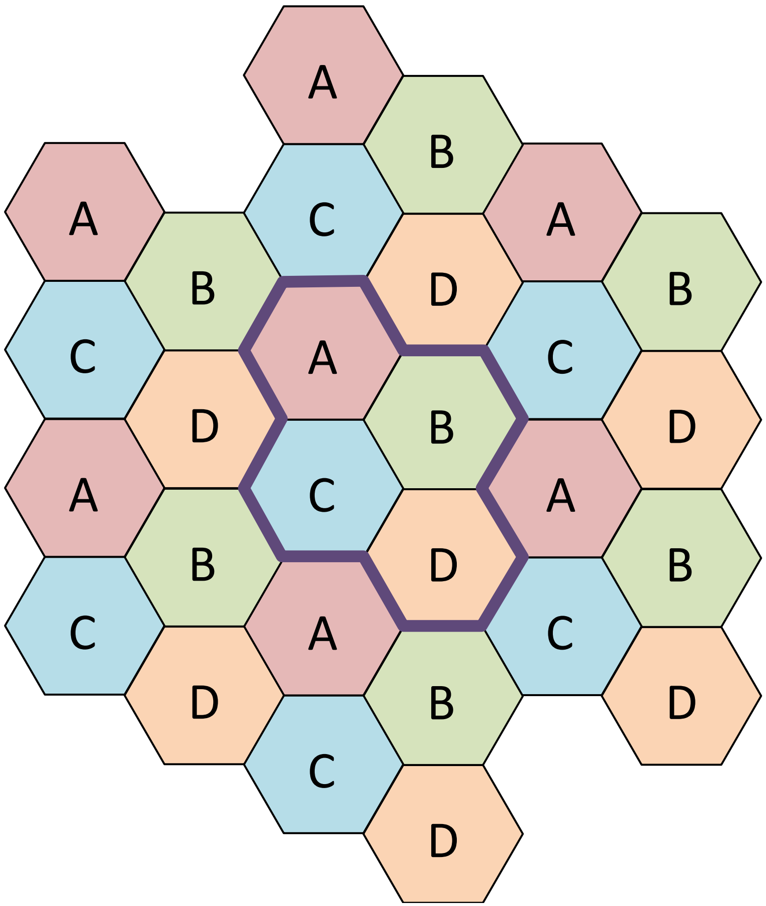


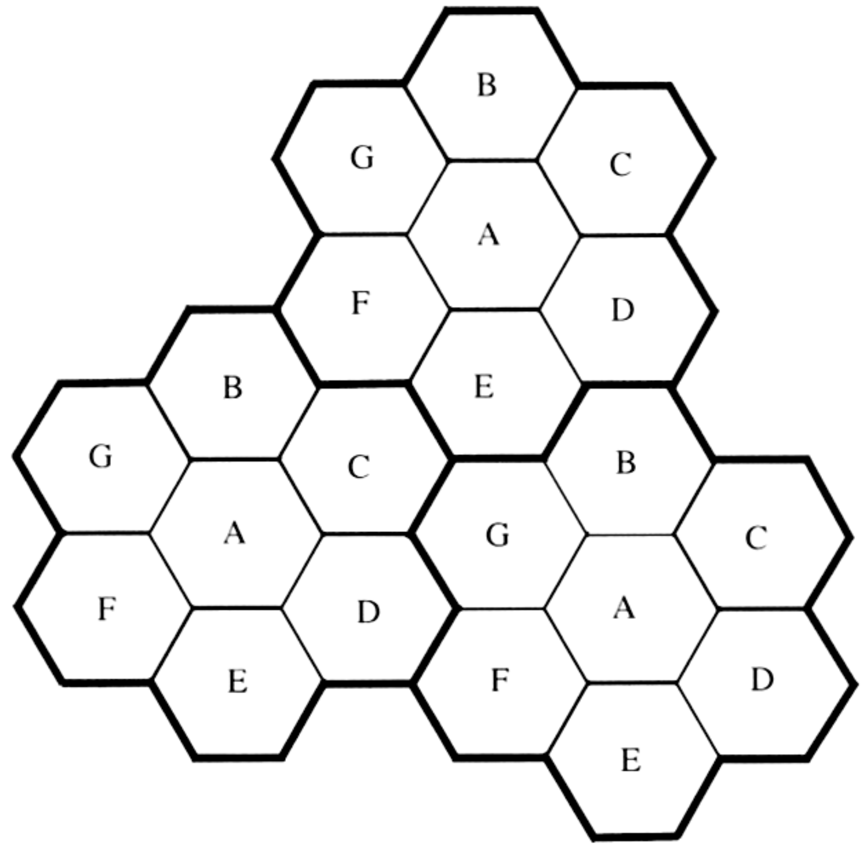
Clusters

- The total coverage area is divided into **clusters.**
- The number of cells (***N***) in a cluster is called the **cluster size**.
- Cells in a cluster collectively use the **complete set** of available frequencies.
- *No* co-channel interference *within* a cluster.
- **Replicated** over the coverage area.
- Example: The picture shows clusters of size $N = 7$, outlined in bold.



Frequency Reuse (N = 4, N = 7)





Frequency reuse factor = 1/N

(Each cell within a cluster is only assigned 1/N of the total available channels in the system.)

“Capacity”

total # available duplex radio channels for the system

cluster size

$$S = mN$$

(**Frequency reuse factor** = 1/N)

#channels allocated to each cell

Ability to handle simultaneous numbers of calls → “**Capacity**”

$$C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times m = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$

total # cells in the system

This formula suggests that if the cluster size N is reduced, more capacity is achieved.*

*Tradeoff: Small value of N may lead to large interference.

Cluster: Summary

A **cluster** is a grouping of cells in which each cell uses different frequencies. A cell's frequencies may be reused by other cells in the system, but those cells will be in other clusters and therefore sufficiently far away not to cause interference.

[Klemens, 2010, p 59]

Cluster size (*N*)

- There are only certain cluster sizes and cell layouts which are possible [Mac Donald,1979].
- *N* can only have values which satisfy

$$N = i^2 + i \times j + j^2$$

where i and j are ***non-negative*** integers.

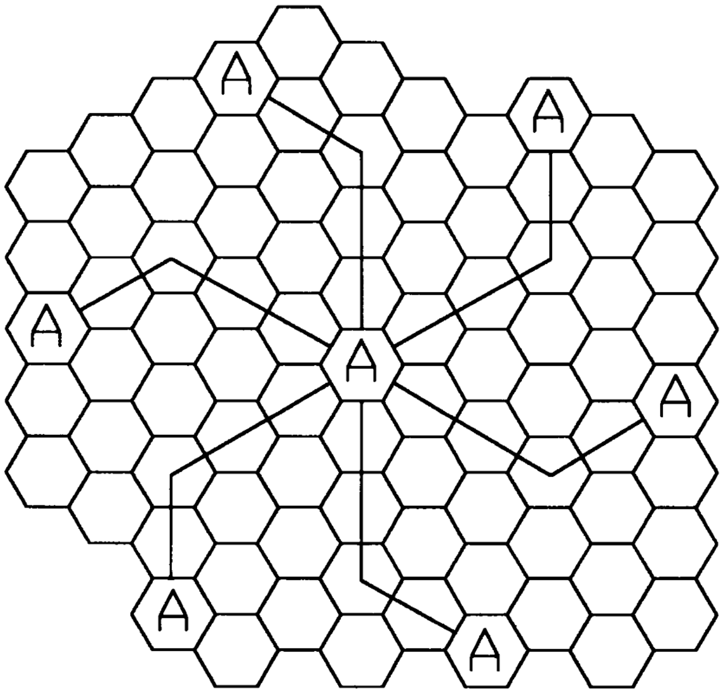
	Cluster Size (*N*)
$i = 1,\ j = 1$	3
$i = 1,\ j = 2$	7

- **Exercise**: For $N = 4$, what are the values of i and j?

Locating co-channel cells

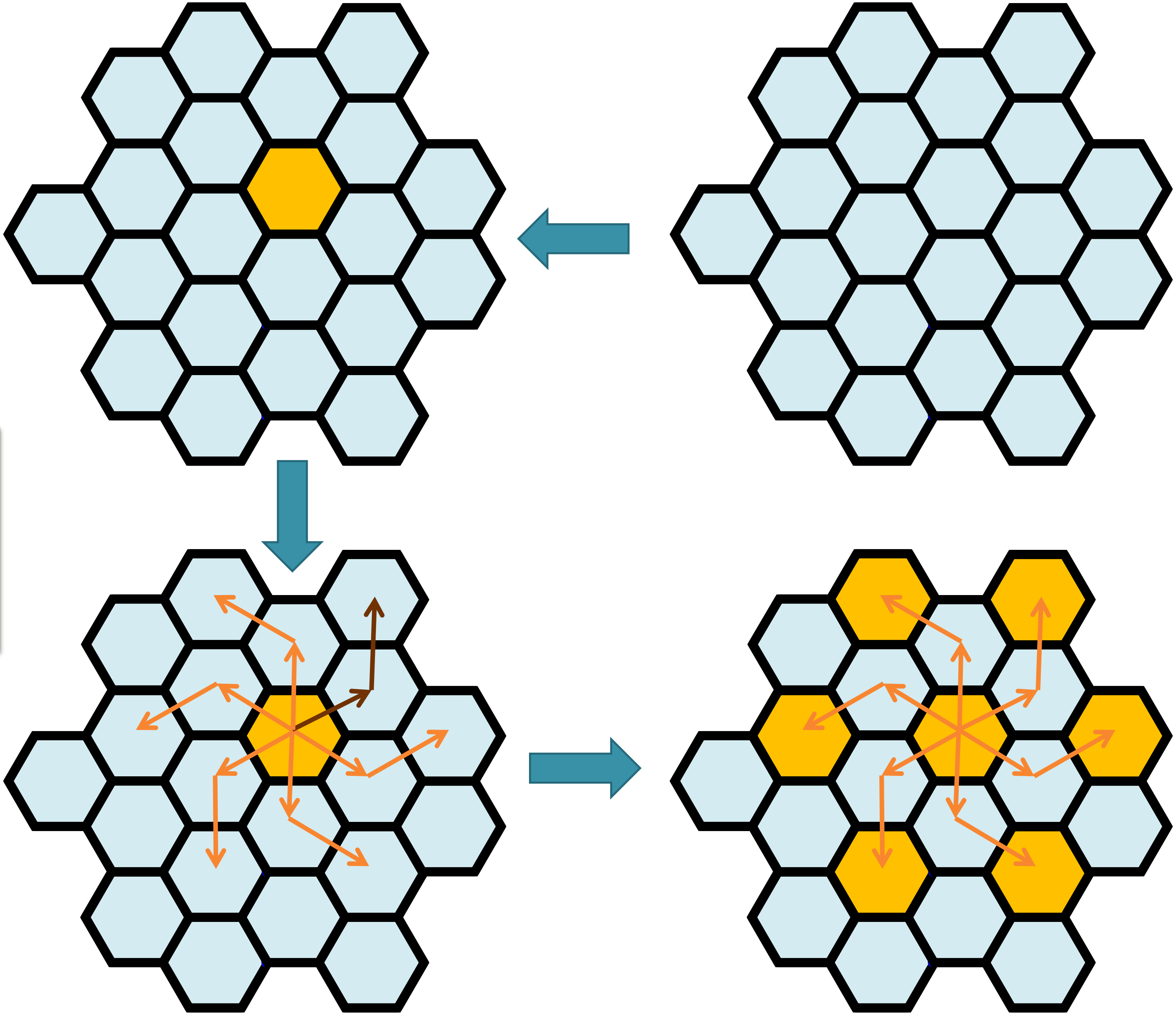
[Rappaport, 2002, p 60]
[Goldsmith, 2005, p 476]

- To locate the **nearest co-channel neighbors** of a particular cell,
  - move i cells along any chain of hexagons and then
  - turn 60 degrees **counter-clockwise** and move j cells.
- Try $N = 19$
  - i = 3
  - j = 2



[Rappaport, 2002, Fig. 3.2]
[Goldsmith, 2005, Fig 15.6]

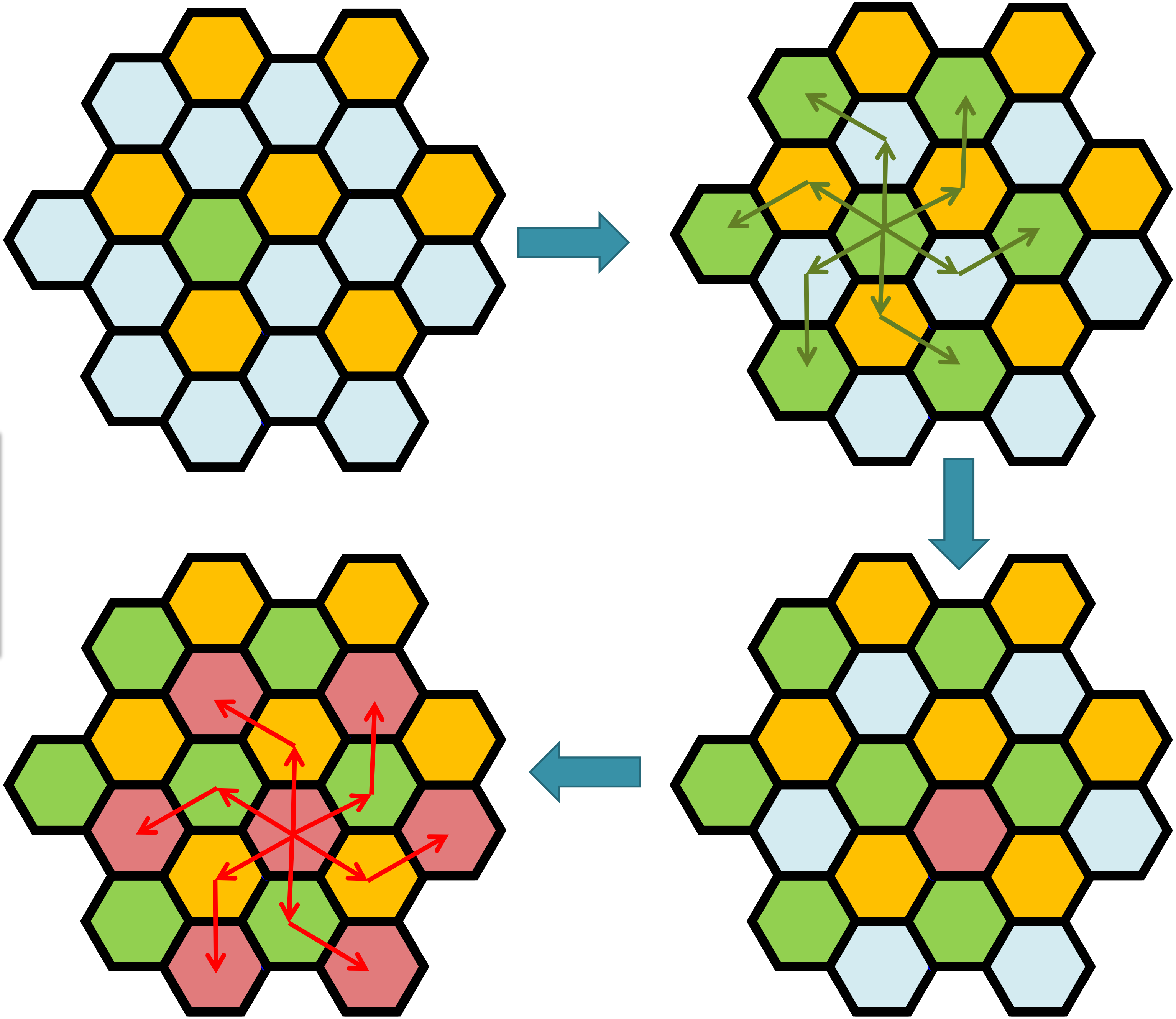
$$3^2 + 2 \cdot 3 + 2^2 = 9 + 6 + 4 = 19$$



$N = 3$

$i = 1$

$j = 1$



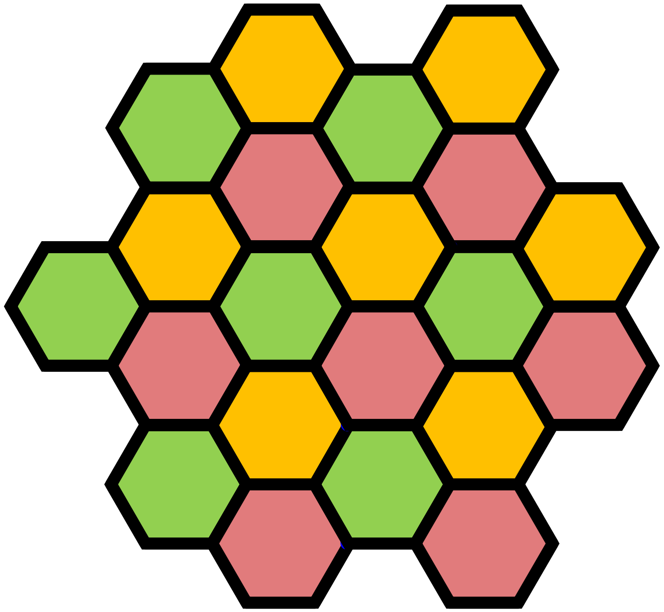
$N = 3$

$i = 1$

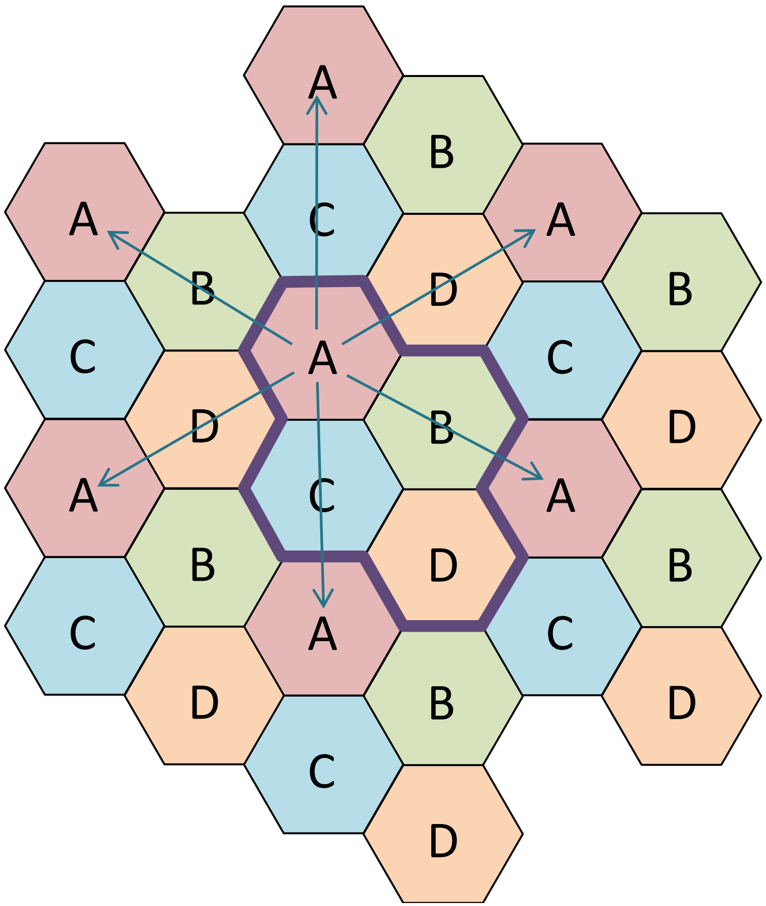
$j = 1$

Locating co-channel cells (N = 3)

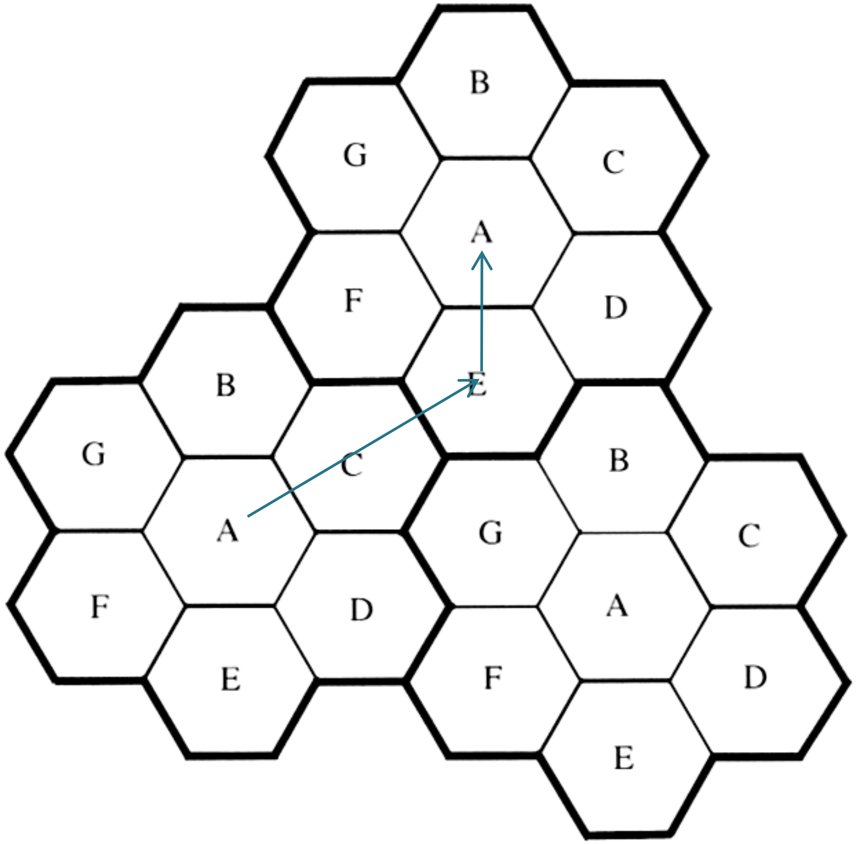
- To locate the nearest co-channel neighbors of a particular cell,
  - move *i* cells along any chain of hexagons and then
  - turn 60 degrees counter-clockwise and move *j* cells.



Locating co-channel cells (N = 4, N = 7)

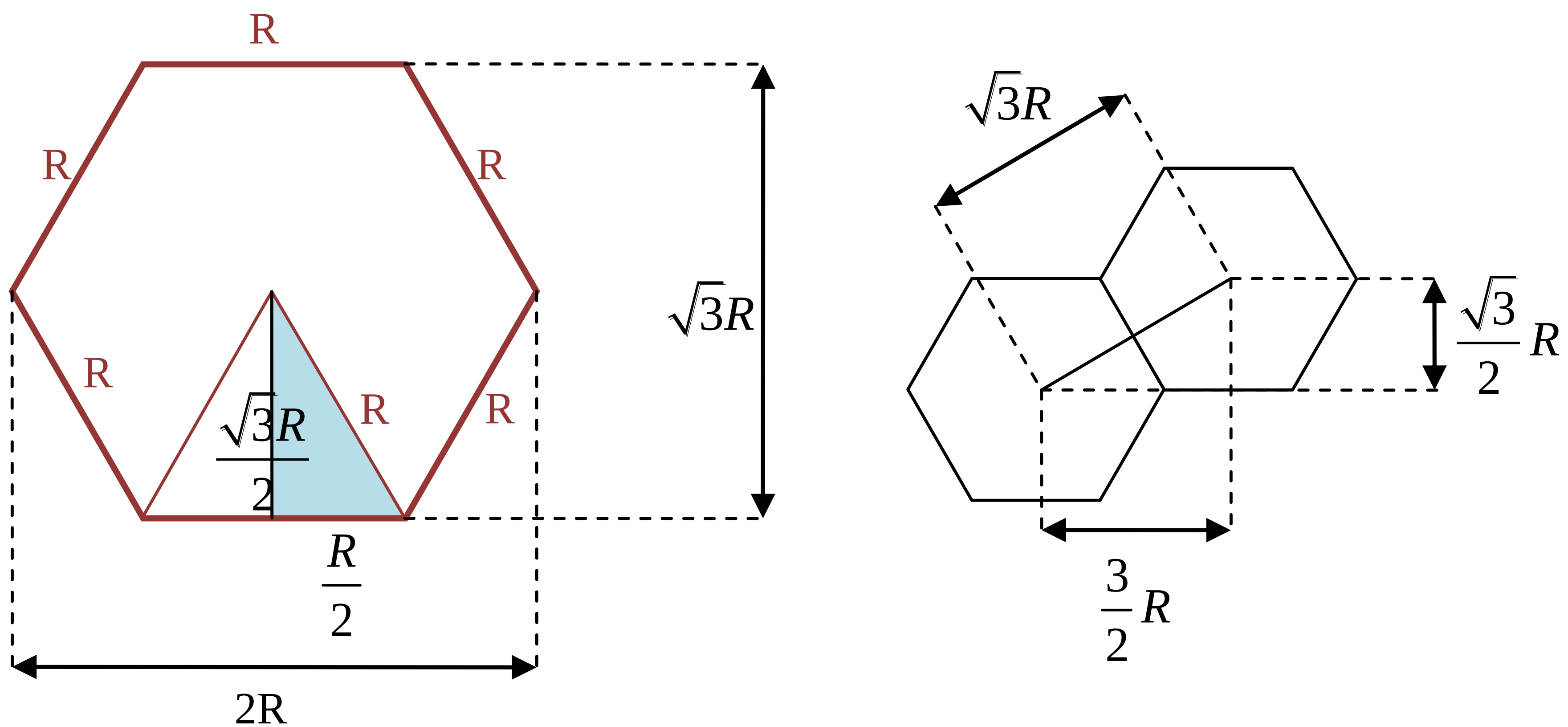


(i = 2, j =0)



(i = 2, j =1)

Hexagon



$$\text{Area} = 6 \times 2 \times \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} R \times \frac{1}{2} R \right) = \frac{3\sqrt{3}}{2} R^2 \approx 2.598 R^2$$

ECS455 Chapter 2

Cellular Systems

2.2 Co-Channel Interference

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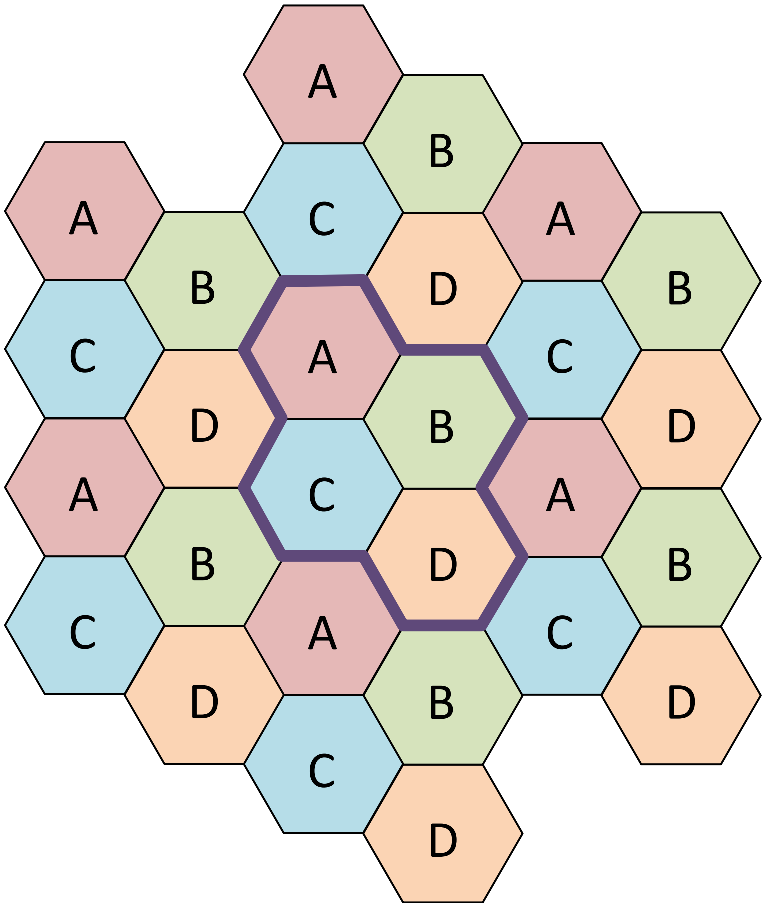
(Intercell)

Co-Channel Interference

- Frequency reuse → co-channel interference
- Consider only nearby interferers.
  - Power decreases rapidly as the distance increases.
- In a **fully equipped hexagonal-shaped** cellular system, there are always K = 6 cochannel interfering cells in the **first tier**.



Three Measures of Signal Quality

- Old (**noise-limited** systems) Signal-to-noise ratio $\text{SNR} = \frac{P_r}{P_{\text{noise}}}$

- Consider both noise & interference — signal-to-interference-plus-noise ratio $\text{SINR} = \frac{P_r}{P_{\text{interference}} + P_{\text{noise}}}$

- The best cellular system design places users that share the same channel at a separation distance (as close as possible) where the intercell interference is just below the maximum tolerable level for the required data rate and BER. Why?

- Good cellular system designs are interference-limited, meaning that the interference power is much larger than the noise power. Signal-to-interference ratio

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}}$$

“Reliable”/“tolerable”?

(Why not as far as possible?)

Co-channel cells, must be spaced **far enough** apart so that interference between users in co-channel cells does not degrade **signal quality** below tolerable levels.

Subjective tests found that people regard an FM signal using a 30 kHz channel bandwidth to be clear if the signal power is at least **sixty times** higher than the noise/interference power.

[Klemens, 2010, p 54]

$$10\log_{10} 60 = 17.78 \approx 18 \text{ dB}$$

We will soon revisit and use these numbers for some more specific calculations

Review: Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma} \Rightarrow P_r = \frac{P_t K d_0^{\gamma}}{d^{\gamma}} \propto \frac{1}{d^{\gamma}}$$

k

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.
- γ is the **path loss exponent**.
  - 2 in free-space model
  - 4 in two-ray model [Goldsmith, 2005, eq. 2.17]

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

[Goldsmith, 2005, Table 2.2]

Not the same as the K shown on the previous slide

SIR (S/I): Definition/Calculation

- K = # co-channel interfering cells
- The **signal-to-interference ratio** (S/I or SIR) for a mobile receiver which monitors a ***forward channel*** can be expressed as

$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

- S = the desired signal **power** from the desired base station
- I_i = the interference **power** caused by the *i*th interfering co-channel cell base station.
- Often called the **carrier-to-interference ratio**: CIR.

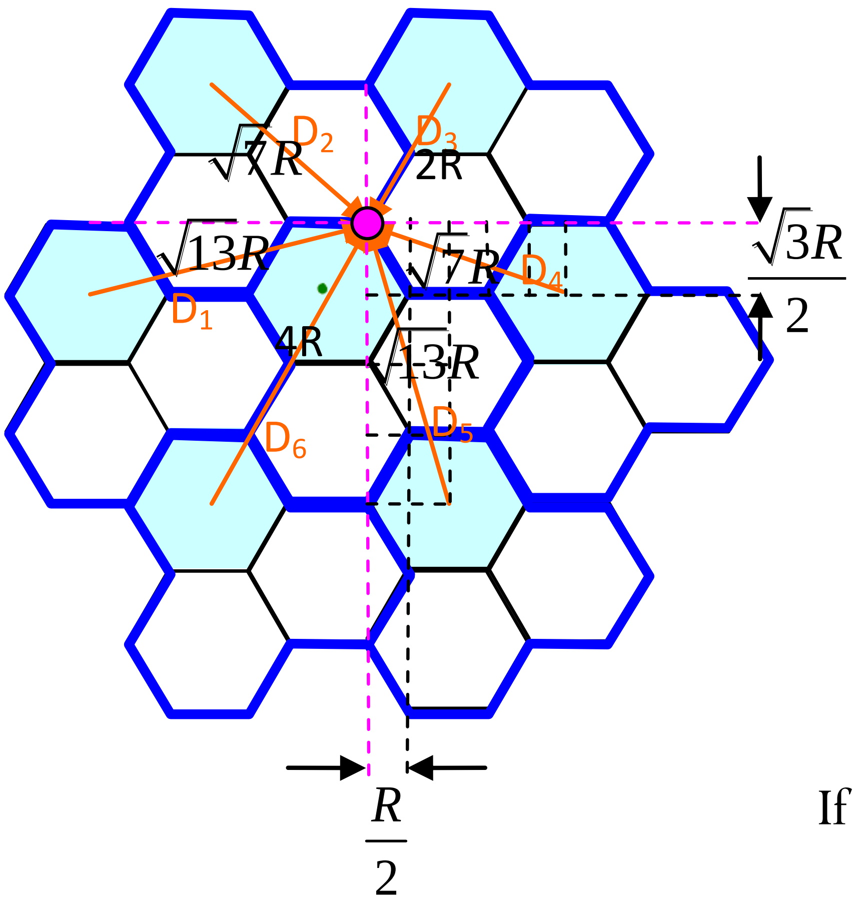
SIR Threshold

[Schwartz, 2005, p 64]

- The SIR should be greater than a specified threshold for proper signal operation.
- In the 1G **AMPS** system, designed for **voice** calls, the threshold for acceptable voice quality is SIR equal to **18 dB**.
- For the 2G digital AMPS system (D-AMPS or IS-54/136), a threshold of 14 dB is deemed suitable.
- For the **GSM** system, a range of **7–12 dB**, depending on the study done, is suggested as the appropriate threshold.
- The probability of error in a digital system depends on the choice of this threshold as well.

SIR: N = 3

- Consider only first tier.
- Worse-case distance

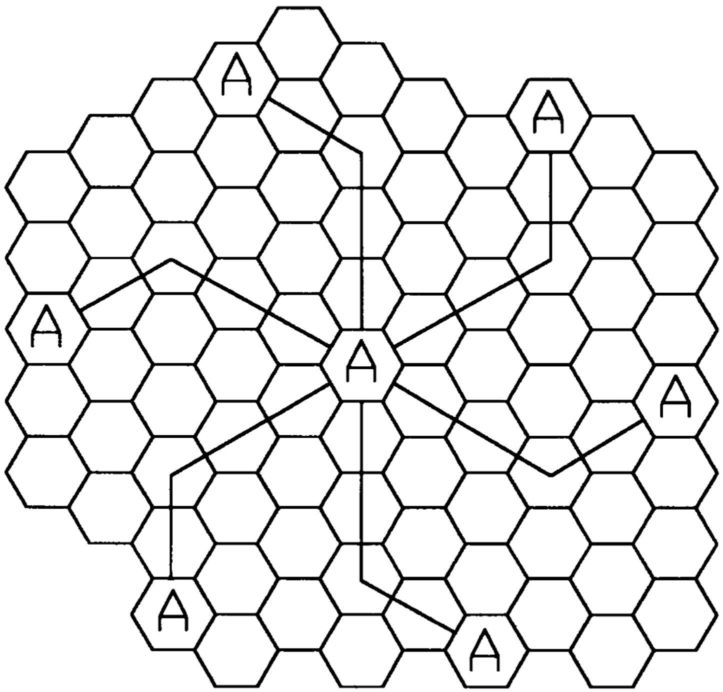


$$\mathrm{SIR} \approx \frac{k/R^{\gamma}}{\sum_i k/D_i^{\gamma}} = \frac{1}{\sum_i 1\Big/\left(\frac{D_i}{R}\right)^{\gamma}} = \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

$$= \frac{1}{2\left(\sqrt{7}\right)^{-\gamma} + 2\left(\sqrt{13}\right)^{-\gamma} + 2^{-\gamma} + 4^{-\gamma}}$$

If N = 19, will the SIR be better or worse?

Approximation

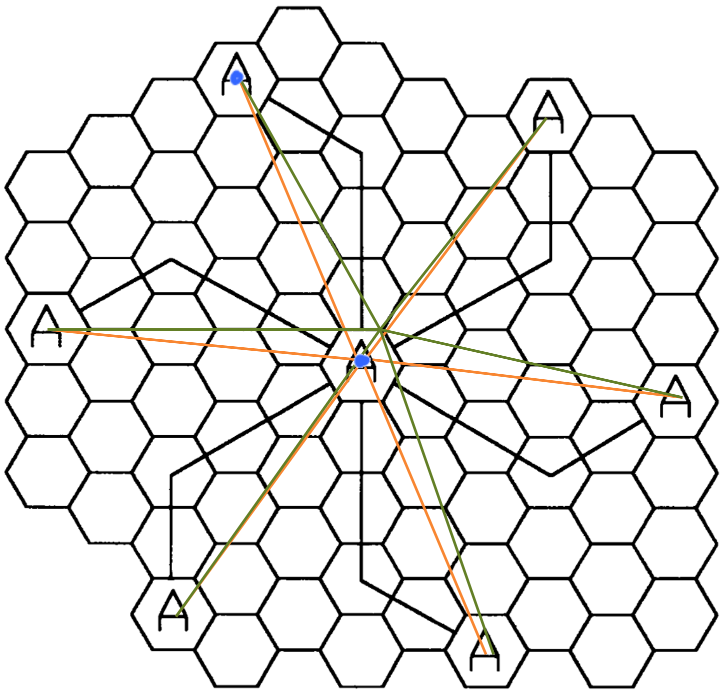


- Consider only first tier.
- Worse-case distance

$$\mathrm{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R} \right)^{-\gamma}}$$

- Use the same D for D_i

Approximation



- Consider only first tier.
- Worse-case distance

$$\mathrm{SIR} \approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}}$$

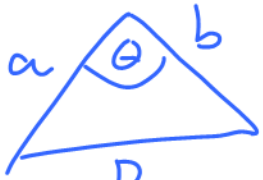
- Use the same D for D_i

$$\mathrm{SIR} \approx \frac{1}{\sum_i \left(\frac{D}{R}\right)^{-\gamma}} \approx \frac{1}{K\left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma}$$

important quantity.

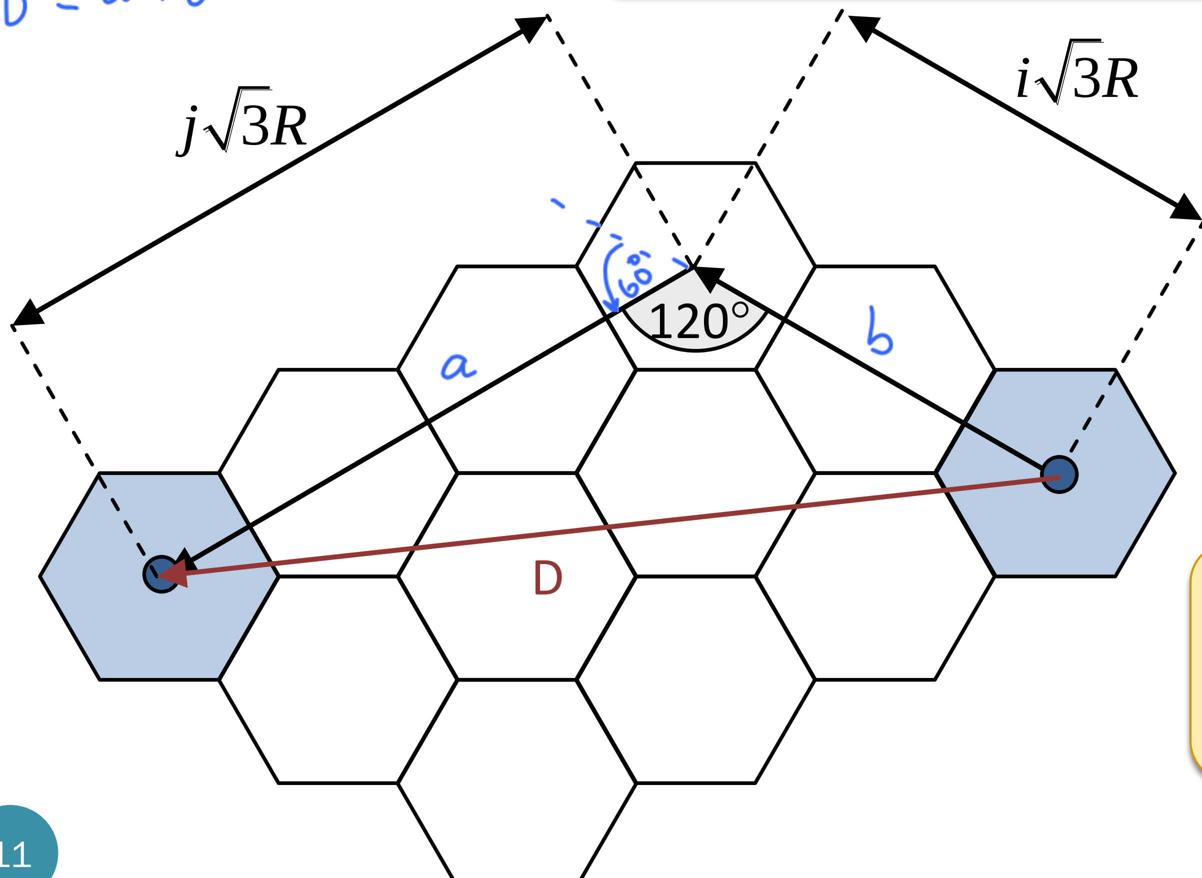
reuse distance D

Center-to-center distance (D)



$D^2 = a^2 + b^2 - 2ab\cos(\theta)$

$$D = \sqrt{\left(i\sqrt{3}R\right)^2 + \left(j\sqrt{3}R\right)^2 - 2\left(i\sqrt{3}R\right)\left(j\sqrt{3}R\right)\cos\left(120^\circ\right)}$$
$$= R\sqrt{3\left(i^2 + j^2 + ij\right)} = R\sqrt{3N}$$



This distance, D, is called **reuse distance**.

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

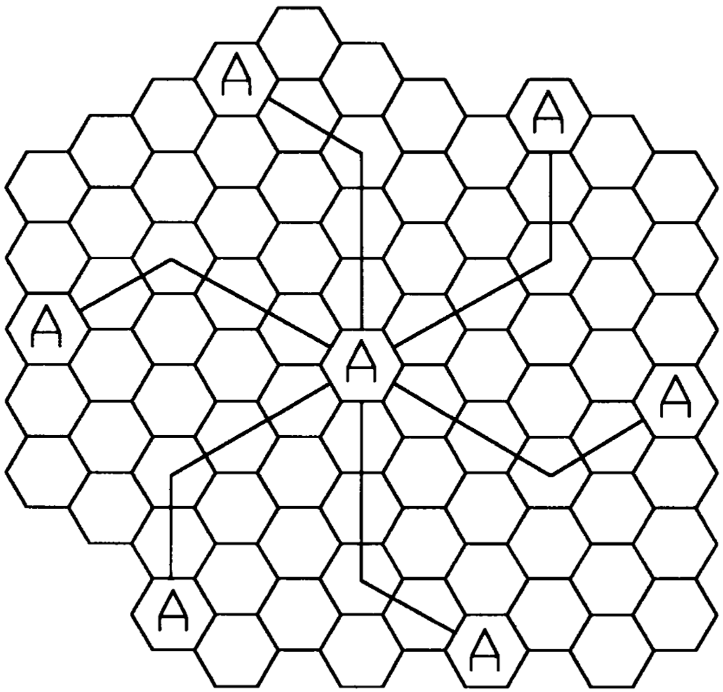
Q and N

Co-channel reuse ratio

$$Q = \frac{D}{R} = \sqrt{3N}.$$

	Cluster Size (N)	Co-channel Reuse Ratio (Q)
$i = 1, j = 1$	3	3
$i = 1, j = 2$	7	4.58
$i = 0, j = 3$	9	5.20
$i = 2, j = 2$	12	6

Approximation: Crude formula



$$\text{SIR} = \frac{P_r}{P_{\text{interference}}} = \frac{P_r}{\sum_{i=1}^{K} P_{\text{of the } i^{th} \text{ interferer}}}$$

$$\approx \frac{1}{\sum_i \left(\frac{D_i}{R}\right)^{-\gamma}} \approx \frac{1}{K\left(\frac{D}{R}\right)^{-\gamma}} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma}$$

$$= \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

As the cell cluster size (N) increases, the spacing (D) between interfering cells increases, reducing the interference.

Summary: Quantity vs Quality

S = total # available duplex radio channels for the system

Path loss exponent

"Capacity" $C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$

Tradeoff

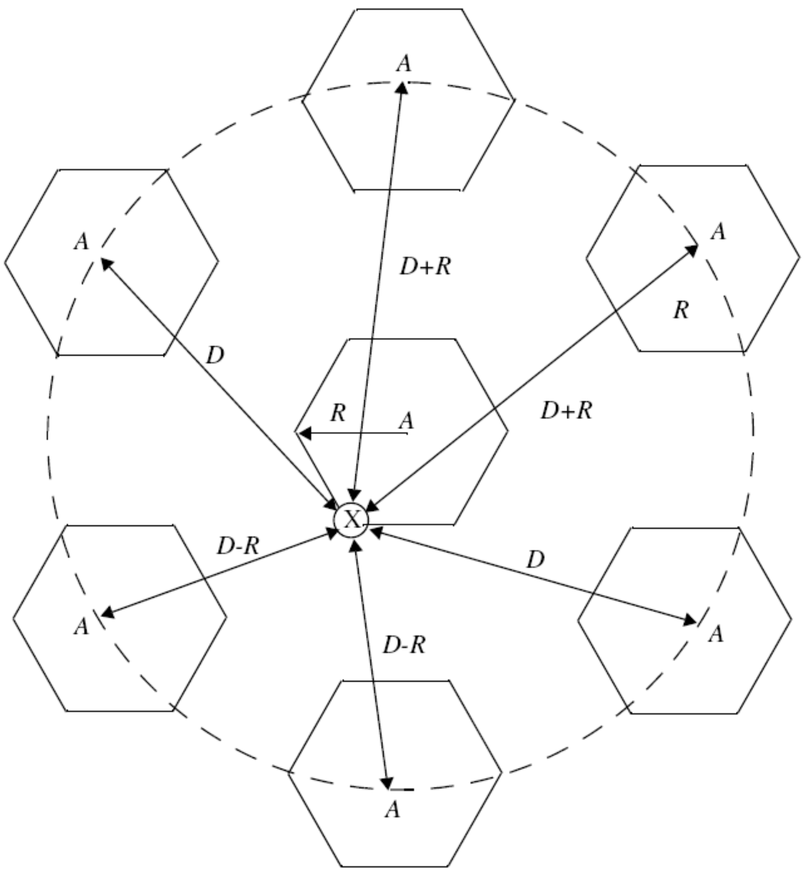
$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$

m = # channels allocated to each cell.

Frequency reuse with **cluster size *N***

This is only half of the big picture.

SIR: N = 7



More accurate calculation…

$$\frac{S}{I} \approx \frac{R^{-4}}{2(D-R)^{-4} + 2(D+R)^{-4} + 2D^{-4}}$$

$$\frac{S}{I} \approx \frac{1}{2(Q-1)^{-4} + 2(Q+1)^{-4} + 2Q^{-4}}$$

ECS455 Chapter 2

Cellular Systems

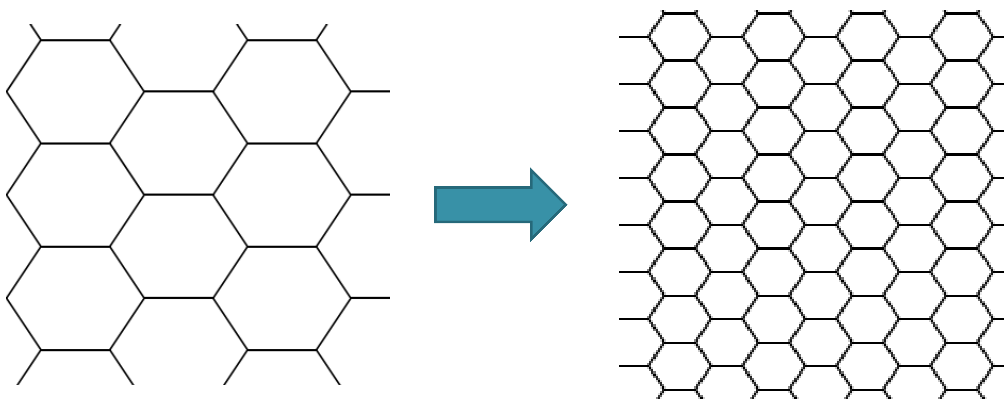
2.3 Sectoring

Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

Improving Coverage and Capacity

- As the demand for wireless service increases, the number of channels assigned to a cell eventually becomes insufficient to support the required number of users.
- At this point, cellular design techniques are needed to provide more channels per unit coverage area.
- Easy!?

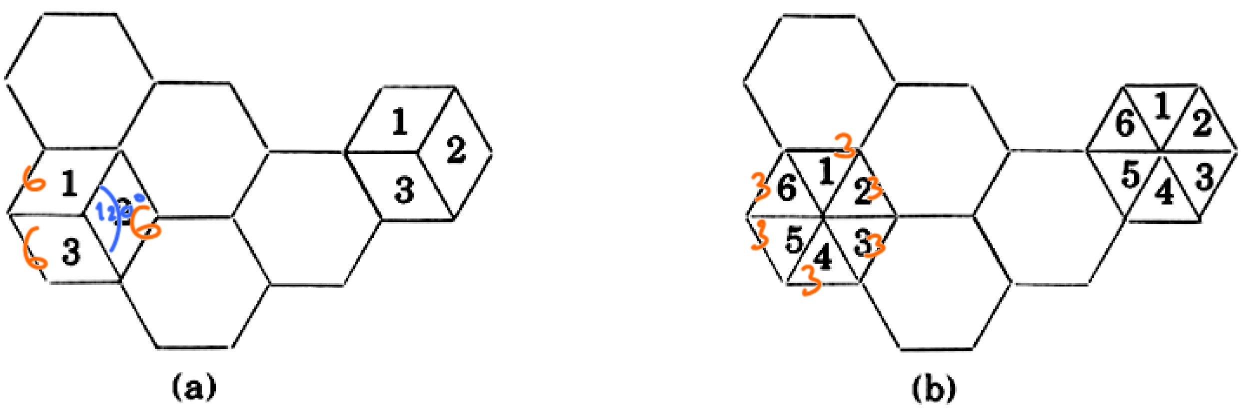
$$C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$



If cells can be reduced in size, more of them can be added in a given area, increasing the overall capacity.

Sectoring (N = 7)

- Ex. With no sectoring, suppose $m = 18$ channels/cell
  - With 120° sectoring, we have 6 channels/sector
  - With 60° sectoring, we have 3 channels/sector
- "Can support the same number of users" per cell
  - In the next section, we will consider different kind of capacity. For such capacity, sectoring will give less capacity.



[Rappaport, 2002]

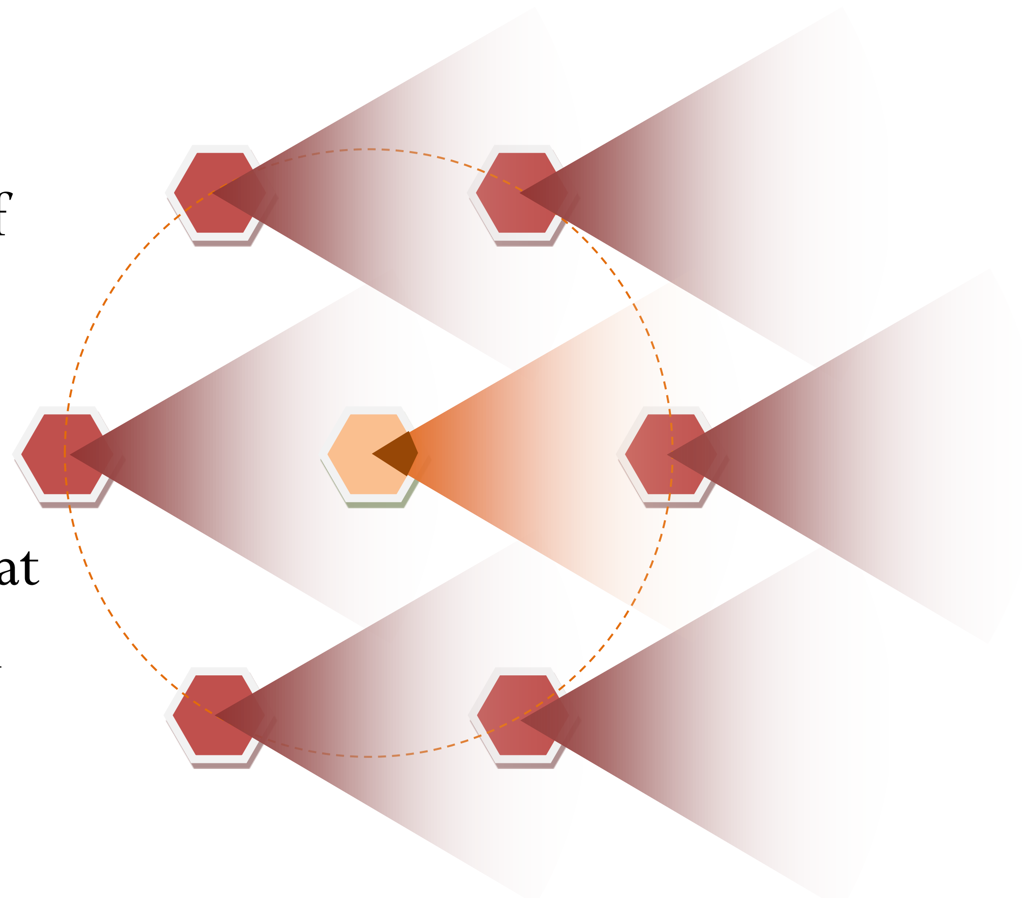
Figure 3.10 (a) 120° sectoring; (b) 60° sectoring.

- Why is this better?

$$\mathrm{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

60 Degree Sectoring

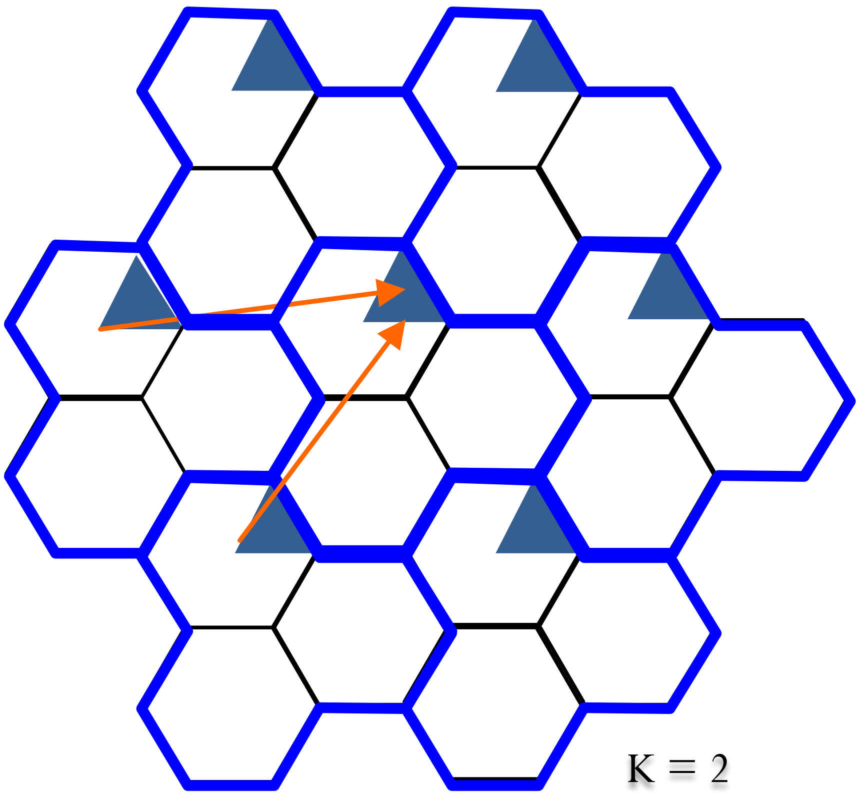
- Out of the 6 co-channel cells in the first tier, only one of them interfere with the center cell.
- If omnidirectional antennas were used at each base station, all 6 co-channel cells would interfere the the center cell.

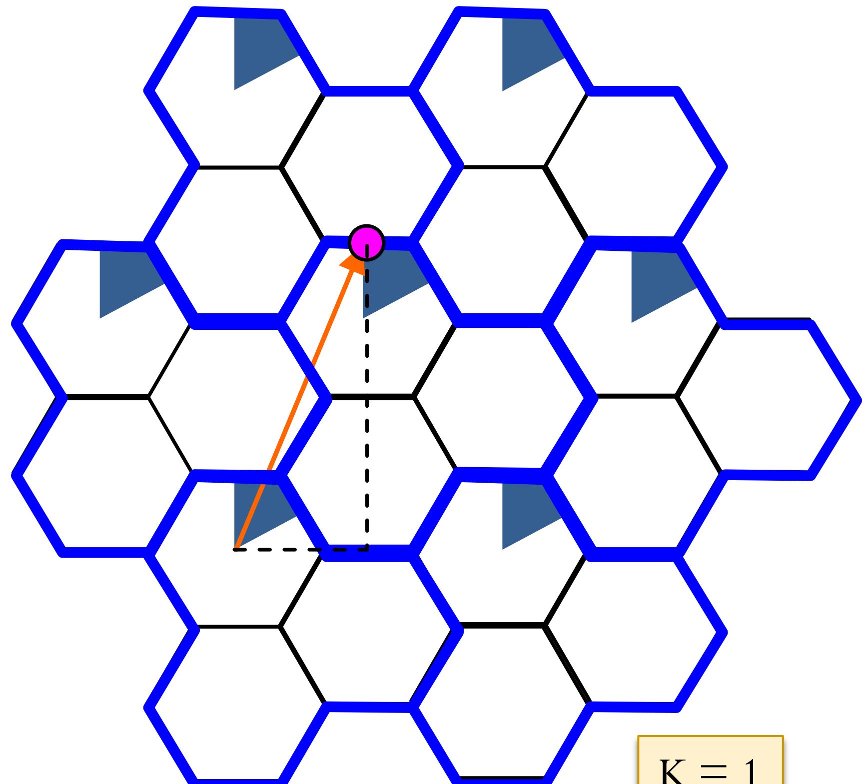


The value of K changes from 6 to 1!

Sectoring (N = 3 , 60°)

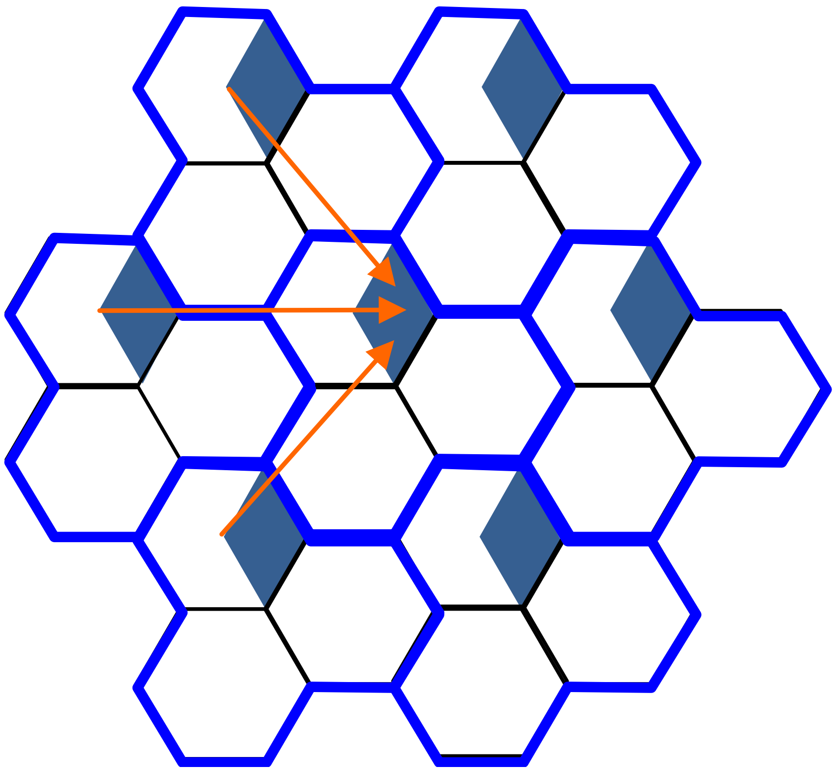
$$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$



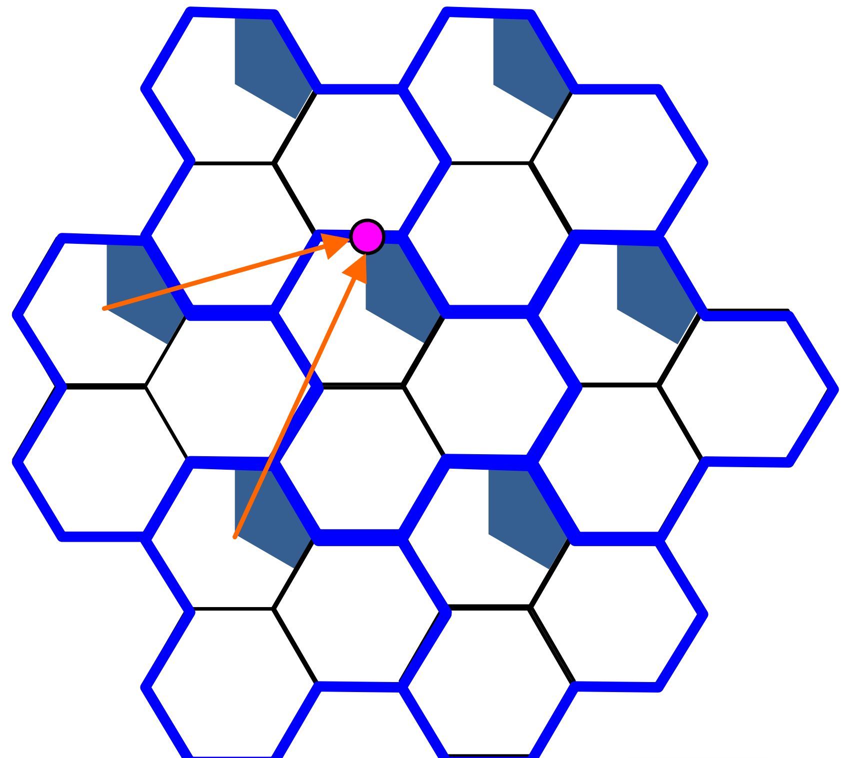


$$SIR \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

Sectoring (N = 3, 120°)



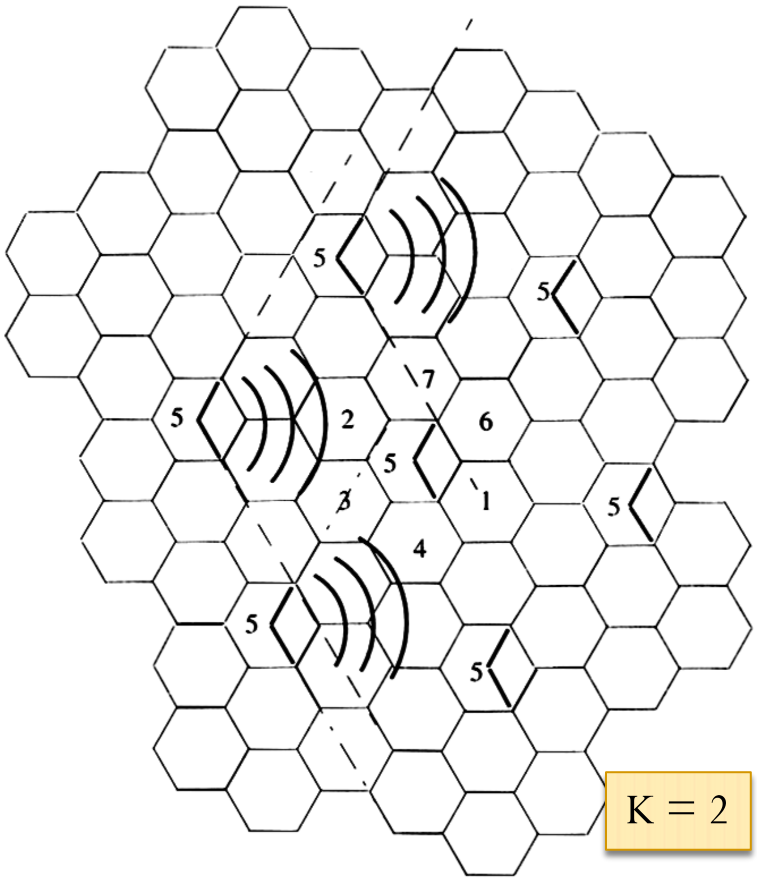
K = 3



K = 2

Sectoring (N = 7, 120°)

Assuming seven-cell reuse, for the case of 120° sectors, the number of interferers in the first tier is reduced from six to two.



[Rappaport, 2002, Fig 3.11]

$$\mathrm{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma} \qquad C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$

K=6 omni
K=1 60°
K=2 120°

Sectoring

- Advantages
  - Reduce interference by reducing K
    - Increase SIR (better call quality).
    - The increase in SIR can be **traded** with reducing the cluster size (N) which increase the capacity.
- Disadvantages
  - Increase number of antennas at each base station.
  - Next section: Decrease **trunking efficiency** due to channel sectoring at the base station.
    - The available channels in the cell must be subdivided and dedicated to a specific antenna.

ECS455 Chapter 2

Cellular Systems

2.4 Traffice Handling Capacity and Erlang B Formula

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Wednesday 15:30-16:30
Friday 9:30-10:30

Capacity Concept: A Revisit

- Q: If I have *m* channels per cell, is it true that my cell can support only *m* user?
- A: Yes and No
- Let's try one example.
- How often do you make a call?
  - 3 calls a day, on average. ← λ_u ← user
- How long is the call?
  - 10 mins (per call), on average. ← $H = \frac{1}{\mu}$
- So, one person uses

$$3\ \frac{\text{calls}}{\text{day}} \times \frac{10\ \text{mins}}{\text{call}} = \frac{30\ \text{mins}}{\text{day}} = \frac{30\ \text{mins}}{24 \times 60\ \text{mins}} = \frac{1}{48}\ [\text{Erlang}]$$

Capacity Concept: A Revisit

- If we can "give" the time that "User 1" is idle to other users,
  - then one channel can support 48 users!!

    (48x "capacity")
- True? (Not quite)

- 48 users is possible if we have a way to manipulate all 48 users to not make calls when another user is using the channel.
- Real users access the channel randomly. (Call initiation/request times are random.)
- If we allow >1 users, then we (the users) will have to deal with congestion.

New Concepts

- Using *m* as the capacity of a cell is too small.
- We can let more than one user share a channel by using it at different times.
- The number of users that a cell can support can then exceed *m*.
- Call initiation times are random
- **Blocked calls**
- Probability of (call) blocking P_b
  - the likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%

Trunking

- Allow a large number (*n*) of users to **share** the relatively small number of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.
- Exploit the **statistical behavior** of users.
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**. In our example, one user utilizes $A_u = \frac{1}{48}$ Erlang
  - Dimensionless
  - Denoted by A. If we have $n=10$ users in the pool, then they utilize $A = \frac{10}{48}$ Erlang.
- **Holding Time**: Average duration of a typical call.
  - Denoted by $\mathbf{H = 1/\mu}$. = 10 mins
- **Request Rate**: The average number of call requests per unit time. Denoted by λ. $\lambda_u = 3$ calls/day, $\lambda = 10 \times 3 = 30$ calls/day.
- Use $\mathbf{A_u}$ and λ_u to denote the corresponding quantities for one user.
- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where n (10) is the number of users supported by the pool (trunked channels) under consideration.

Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion. Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).

  P_b $\quad P_b \leq 0.02$

  - The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

M/M/m/m Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration of the time that a user occupies a channel is** ***exponentially distributed***, so that longer calls are less likely to occur.
- There are m channels available in the trunking pool.
  - For us, m = the number of channels for a cell (C) or for a sector

Erlang B Formula

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

m = Number of trunked channels

Call blocking probability

$A =$ traffic intensity or load [Erlangs]

$= \frac{\lambda}{\mu}$

λ = Average # call attempts/requests per unit time

$\frac{1}{\mu} = H$ = Average call length

$= \lambda \times \frac{1}{\mu}$

$= \lambda \times H$

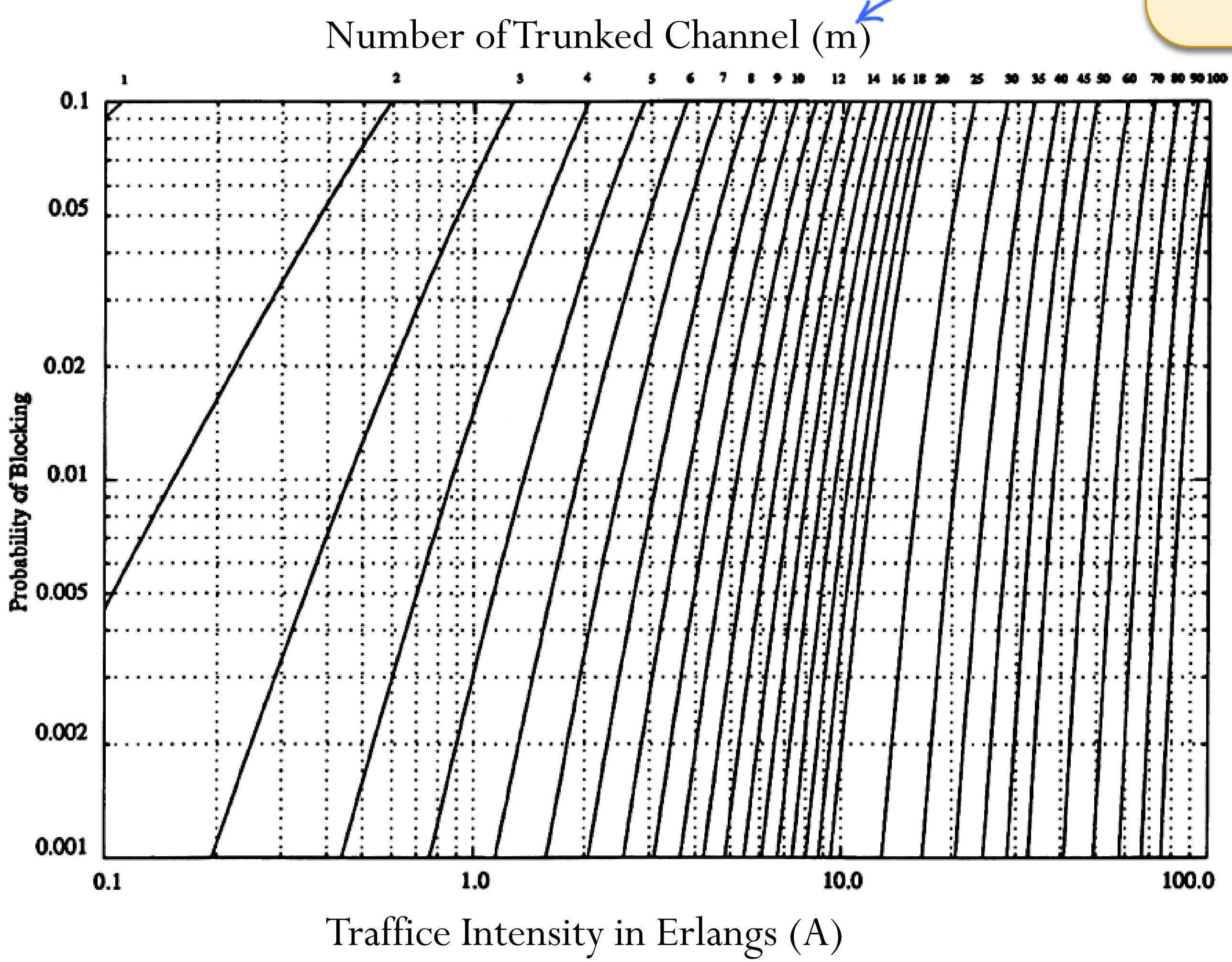
In MATLAB, use `erlangb(m,A)`

not a build-in function.

You can download the .m file.

Erlang B Formula and Chart

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$



(log-log plot)

Example 1

$P_b \le 0.005$

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

  (a) 5 $m = 5$

  (b) 10 $m = 10$

- Assume each user generates $A_u = 0.1$ Erlangs of traffic.

For example,

6 times/day

average 24 min

$$A_u = \lambda_u \times \frac{1}{\mu}$$

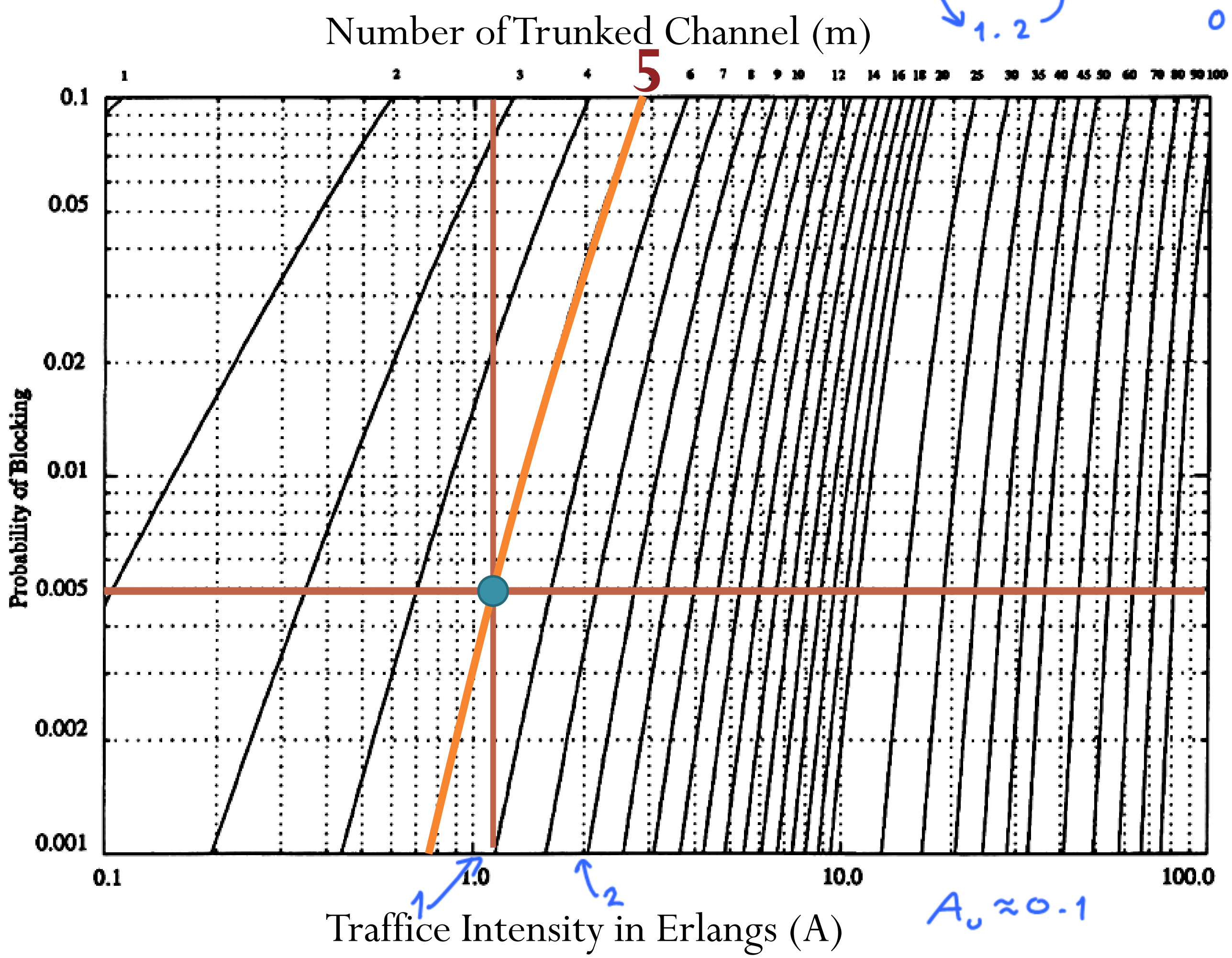
$$\Rightarrow \frac{6 \times 24}{24 \times 60} = \frac{1}{10}$$

$m = 5 \xrightarrow{\text{MATLAB}} A = 1.13 \rightarrow 11$ users

$m = 5$

A	P_b
1.1	0.0045
1.13	0.0050
1.14	0.0051
1.15	0.0053
1.2	0.0063

Example 1a



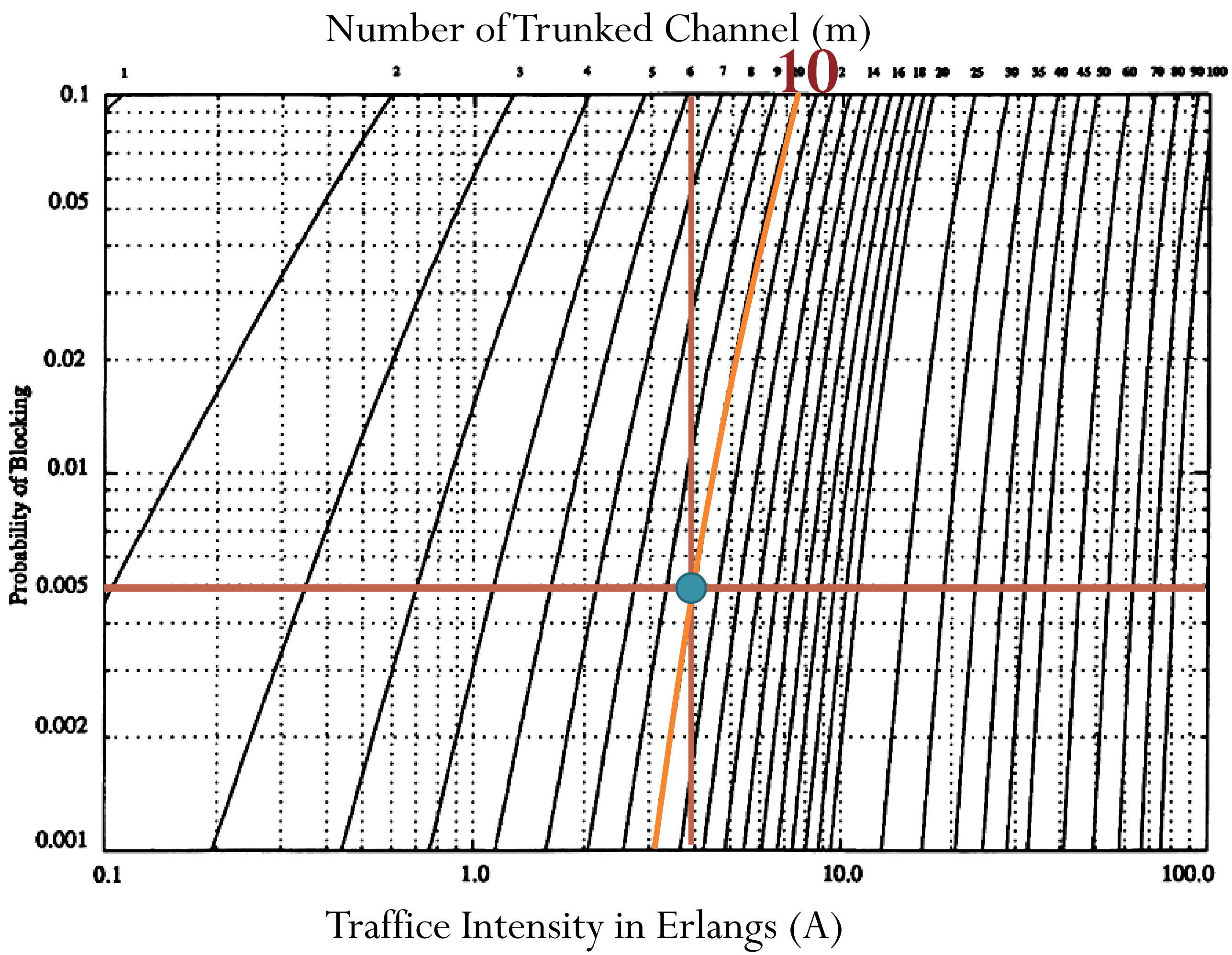
$A_u \approx 0.1$

$A = nA_u$

$A \approx 1 \Rightarrow n \approx 10$ users

Example 1b

m = 10 ⟶ A = 3.96 ⟶ 39 users



$A \approx 4 \Rightarrow n \approx 40$ users

Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes $H = \frac{1}{\mu} = 2 \text{ mins}$
  - the probability of blocking is to be no more than 1%. $P_b \leq 0.01$
- If there are a total of 395 traffic channels for a seven-cell ($N = 7$) reuse system, there will be about 57 ($395/7$) traffic channels per cell. (m)
- From the Erlang B formula, can handle 44.2 Erlangs or **1326 calls per hour**.

$$A = \lambda \times \frac{1}{\mu}$$

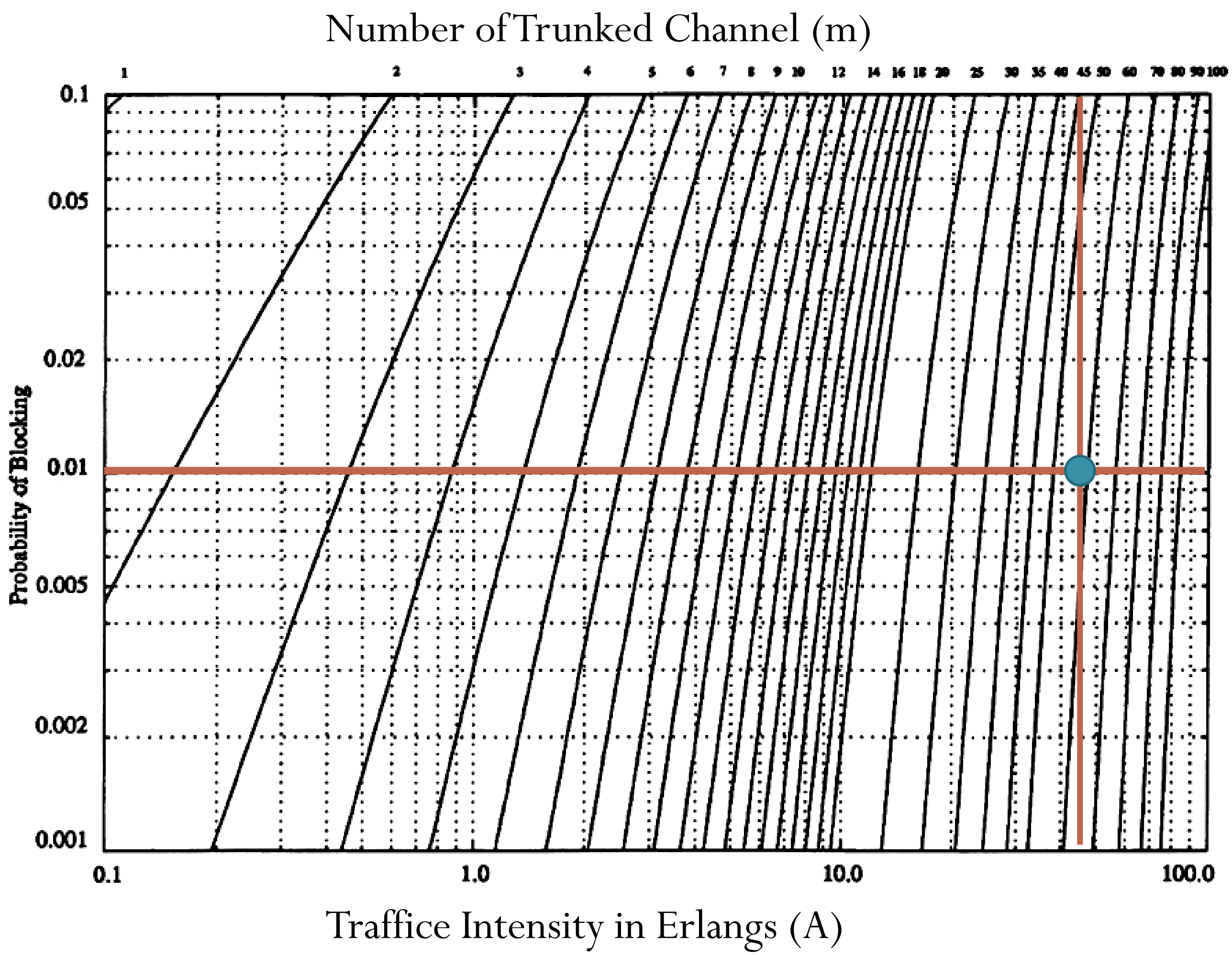
$$44.2 = \lambda \times \frac{2 \text{ mins}}{\text{call}}$$

$$\lambda = \frac{44.2}{2} \frac{\text{calls}}{\text{min}} = 22.1 \text{ calls/min}$$

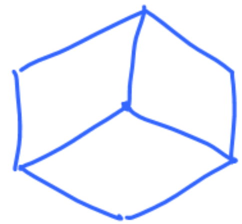
$\times 60$

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.1: Erlang B



Example 2.2

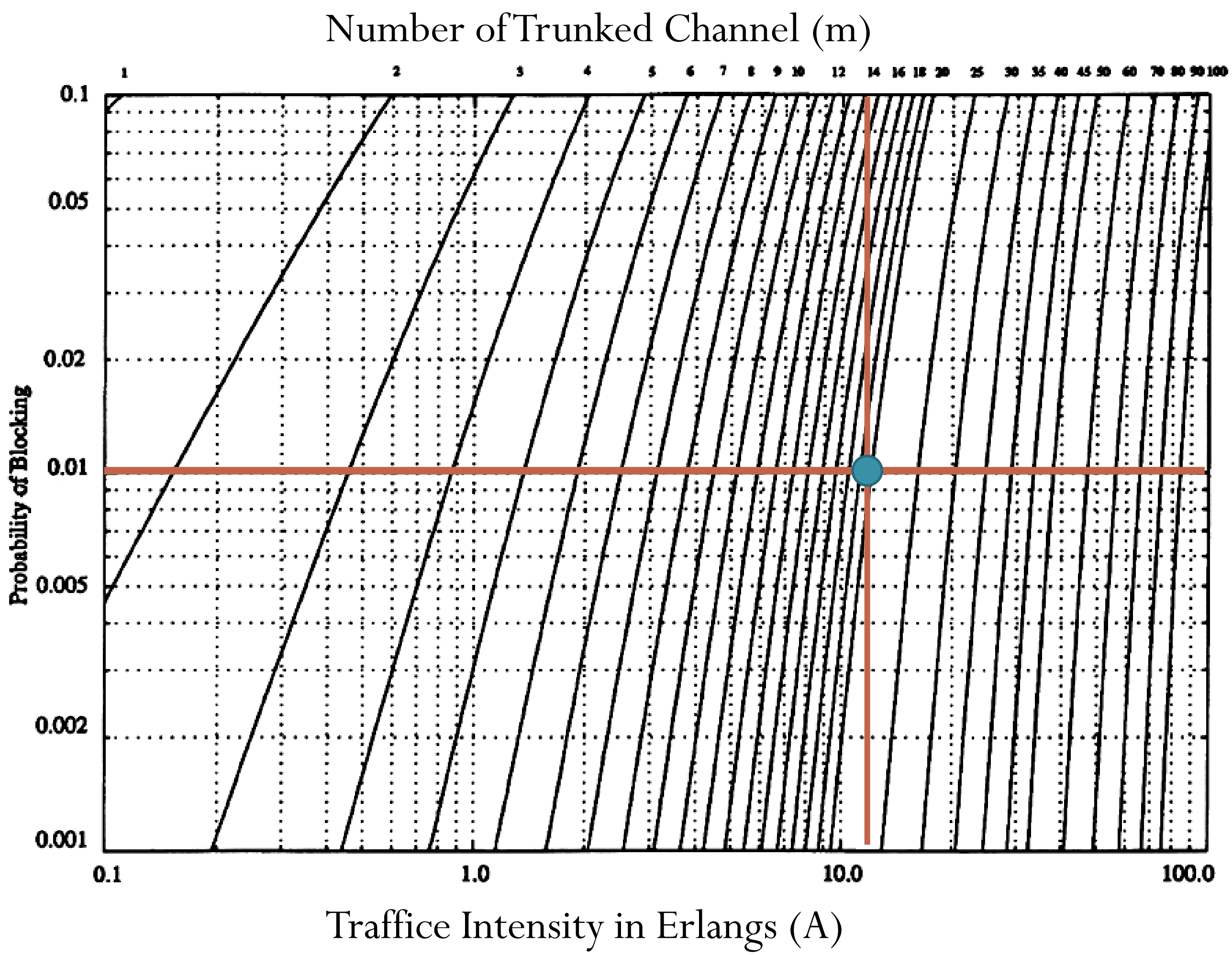


- Now employing **120° sectoring**, there are only 19 channels per sector (57/3 ~~antennas~~ channel).
- For the same probability of blocking ($P_b \le 0.01$) and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 =$ **1008 calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the SIR for each user in the system.

worse Erlang
but
better SIR.

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.2: Erlang B



Erlang B Trunking Efficiency

Table 3.4 Capacity of an Erlang B System

Number of Channels C	Capacity (Erlangs) for GOS 1% = 0.01	= 0.005	= 0.002	0.1% = 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

[Rappaport, 2002, Table 3.4]

Summary of Chapter 2: Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

"Capacity" $C = \frac{A_{total}}{A_{cell}} \times \frac{S}{N}$

Tradeoff

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

Path loss exponent

m = # channels allocated to each cell.

Omni-directional: K = 6

120° Sectoring: K = 2

60° Sectoring: K = 1

Trunking

m = # trunked channels

λ = Average # call attempts/requests per unit time

Call blocking probability

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

A = traffic intensity or load [Erlangs] $= \frac{\lambda}{\mu}$

Erlang-B formula

$\frac{1}{\mu} = H$ = Average call length

Example 3 (1)

- System Design
- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20\times10^6}{2\times25\times10^3} = 400 \text{ channels}$$

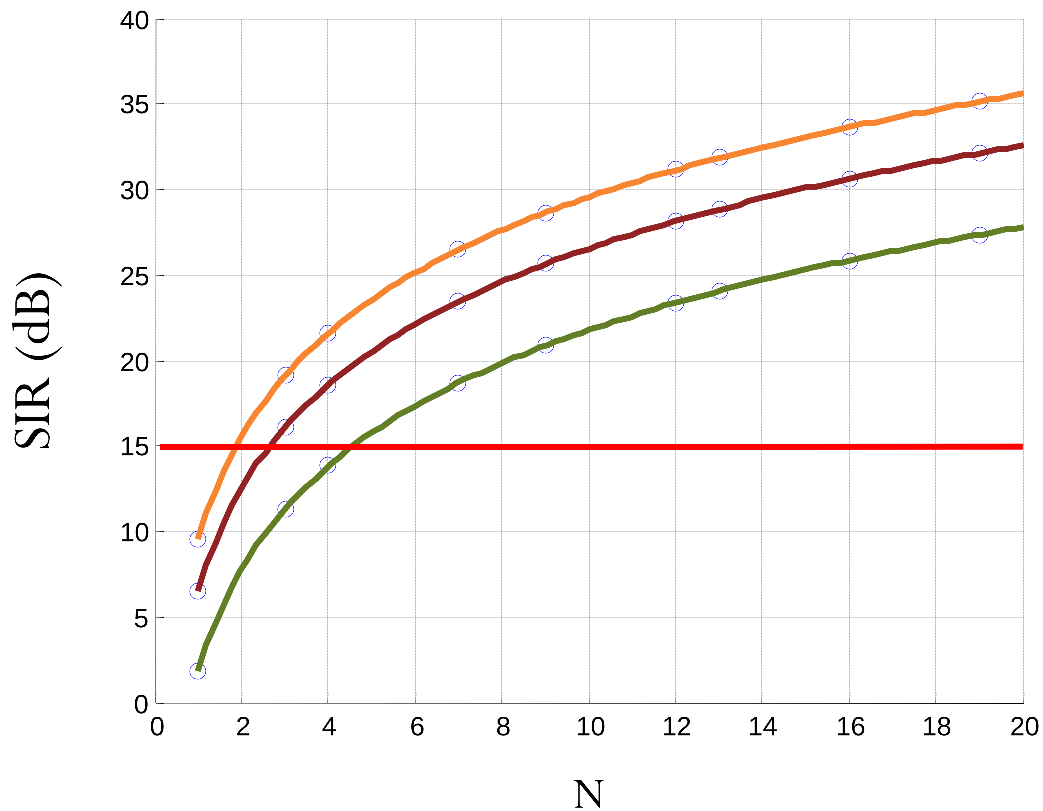
- $\gamma = 4$
- Design requirements:
  - SIR $\geq$ 15 dB
  - $P_b \leq 5\%$

Example 3 (2)

- SIR ≥ 15 dB

$$\frac{S}{I} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
    N = [1, 3, 4, 7, 9, 12, 13, 16, 19];
    SIR = 10*log10(1/K*((sqrt(3*N)).^y))
    plot(N,SIR,'o')
    N = linspace(1,20,100);
    SIR = 10*log10(1/K*((sqrt(3*N)).^y));
    plot(N,SIR)
end
```



60° sectoring
$K = 1 \rightarrow N = 3$

120°
$K = 2 \rightarrow N = 3$

omni
$K = 6 \rightarrow N = 7$

Example 3 (3)

	Omnidirectional	Sectoring (120°)	Sectoring (60°)
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
S=400 #channels/cell	400/7 = 57	400/3 = 133	400/3 = 133
#sectors	1	3	6
m= #channels/sector	57	133/3 = 44	133/6 = 22
A [Erlangs]/sector	51.55	38.56 best	17.13
A [Erlangs]/cell	51.55	38.56×3 = 115.68	17.13×6 = 102.78
#users/cell	18558	41645	37001

Assume that each user makes 2 calls/day and 2 min/call on average → 1/360 Erlangs.

Make sure that you understand where numbers in this table come from!

ECS455 2011/2, Chapter 3 Part 1, Dr.Prapun

call duration ~ $\mathcal{E}(\mu)$

In this note, we will look at the analysis of the call blocking probability in a cellular system under the M/M/m/m assumption. We assume

Poisson arrivals/requests with rate λ

(a) Blocked calls cleared

- No queuing for call requests.
- For every user who requests service, there is no setup time and the user is given immediate access to a channel if one is available.
- If no channels are available, the requesting user is blocked without access and is free to try again later.

(b) Calls arrive as determined by a ***Poisson process***.

(c) Arrivals of requests are memoryless: all users, including blocked users, may request a channel at any time.

(d) There are an infinite number of users (with finite overall request rate).

- The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.

(e) The duration of the time that a user occupies a channel is ***exponentially*** distributed, so that longer calls are less likely to occur.

(f) There are m channels available in the trunking pool.

- For us, m = the number of channels for a cell or for a sector.

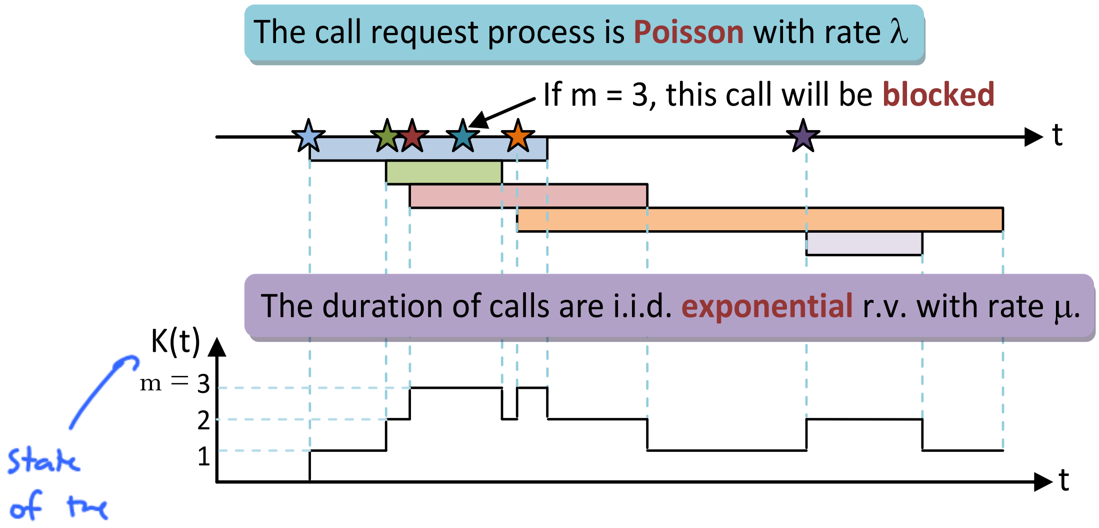


Figure 1: M/M/m/m Assumption

Some of the conditions above are drawn in Figure 1. Later on, we will try to relax some of the assumptions above to make the analysis more realistic. In Figure 1, we also show one important parameter of the system: $K(t)$. This is the number of used channels at time t. When $K(t) < m$, new call can be made. When $K(t) = m$, new call request(s) will be blocked. So, we can find the call blocking probability by looking at the value of $K(t)$. In particular, we want to find out the proportion of time the system has $K = m$.

Poisson process and some probability concepts will be reviewed in Section 1. Most of the probability reviews will be put in footnotes so that they do not interfere with the flow of the presentation.

1 Poisson Process

← call generation/initiation process

In this section, we consider an important random process called ***Poisson process*** (PP). This process is a popular model for customer arrivals or calls requested to telephone systems.

1.1. We start by picturing a Poisson Process as a random arrangement of "marks" (denoted by "x") on the time axis. These marks may indicate the time that customers arrive or the time that call requests are made:

In the language of "queuing theory", the marks denote ***arrival times***.

1.2. In this class, we will focus on one kind of Poisson process called ***homogeneous Poisson process***. Therefore, from now on, when we say "Poisson process", what we mean is "homogeneous Poisson process".

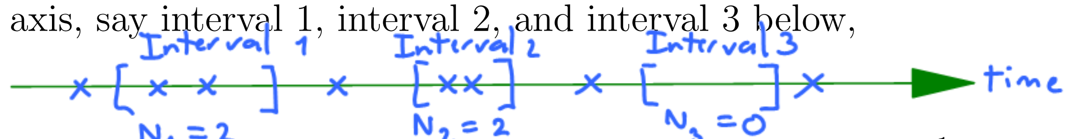
1.3. The first property of Poisson process that you should remember is that

there is only one parameter for Poisson process.

This parameter is the ***rate*** or ***intensity*** of arrivals (the average number of arrivals per unit time.

- We used λ to denote this parameter.
- For homogeneous Poisson process, λ is a constant.
- For non-homogeneous Poisson process, λ is a function of time, say $\lambda(t)$
- Our λ is constant because we focus on homogeneous Poisson process.

1.4. How can λ, which is the only parameter, controls Poisson process? The key idea is that the Poisson process is as random/unstructured as a process can be. Therefore, if we consider many non-overlapping intervals on the time axis, say interval 1, interval 2, and interval 3 below,



and count the number of arrivals N_1, N_2 and N_3 in these intervals[1].

[1]Note that the numbers N_1, N_2, and N_3 are random. Because they are counting the number of arrivals, we know that they can be any non-negative integers:

$$0, 1, 2, 3, \ldots.$$

Because we don't know their exact values, we describe them via the likelihood or probability that they will take one of these possible values. For example, for N_1, we describe it by

$$P[N_1 = 0], P[N_1 = 1], P[N_1 = 2], \ldots$$

where $P[N_1 = k]$ is the probability that N_1 takes the value k. Such list of numbers is a bit tedious. So, we define a function

$$p_{N_1}(k) = P[N_1 = k].$$

This function $p_{N_1}(\cdot)$ tells the probability that N_1 will take a particular value (k). We call p_{N_1} the probability mass function (pmf) of N_1. At this point, we don't know much about $p_{N_1}(k)$ except that its values will be between 0 and 1 and that

$$\sum_{k=0}^{\infty} p_{N_1}(k) = 1.$$

These two properties are the necessary and sufficient conditions for any pmf.

Then, the numbers N_1, N_2 and N_3 in our example above should be independent[2]; for example, knowing the value of N_1 does not tell us anything at all about what N_2 and N_3 will be. This is what we are going to take as a vague definition of the "complete randomness" of the Poisson process.

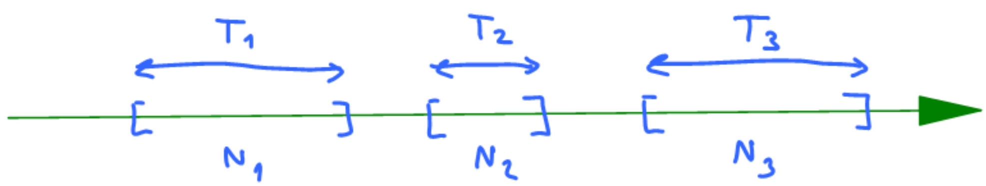
To summarize, now we have one more property of a Poisson process:

> The number of arrivals in non-overlapping intervals are independent.

1.5. Do we know anything else about N_1, N_2, and N_3? Again, we have only one parameter λ for a Poisson process. So, can we connect λ with N_1, N_2, and N_3?

Recall that λ is the average number of arrivals per unit time. So, if $\lambda = 5$ arrivals/hour, then we expect that N_1, N_2, and N_3 should conform with this λ, statistically.

Let's first be more specific about the time duration of the intervals that we have earlier. Suppose their lengths are T_1, T_2, and T_3 respectively



Then, you should expect[3] that

$$\begin{aligned} \mathbb{E}N_1 &= \lambda T_1, \\ \mathbb{E}N_2 &= \lambda T_2, \text{ and} \\ \mathbb{E}N_3 &= \lambda T_3. \end{aligned}$$

---

[2]By saying that something are independent, we mean it in terms of probability. In particular, when we say that N_1 and N_2 are independent, it means that

$$P[N_1 = k \text{ and } N_2 = m]$$

(which is the probability that $N_1 = k$ and $N_2 = m$) can be written as the product

$$p_{N_1}(k) \times p_{N_2}(k)$$

[3]Recall that $\mathbb{E}N_1$ is the expectation (average) of the random variable N_1. Formula-wise, we can calculate $\mathbb{E}N_1$ from

$$\mathbb{E}N_1 = \sum_{k=0}^{\infty} k \times P[N_1 = k];$$

that is the sum of the possible values of N_1 weighted by the corresponding propabilities

For example, suppose $\lambda = 5$ arrivals/hour and $T_1 = 2$ hour. Then you would see about $\lambda \times T_1 = 10$ arrivals during the first interval. Of course, the number of arrivals is random. SO, this number 10 is an average or the expected number, not the actual value.

To summarize, we now know one more property of a Poisson process:

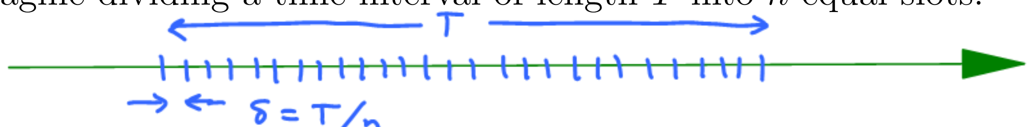
For any interval of length T, the expected number of arrivals in this interval is given by

$$\mathbb{E}N = \lambda T. \tag{1}$$

1.1 Discrete-time (small-slot) approximation of a Poisson process

1.6. The next key idea is to consider a small interval:

Imagine dividing a time interval of length T into n equal slots.



Then each slot would be a time interval of duration $\delta = T/n$. For example, if $T = 20$ hours and $n = 10,000$, then each slot would have length

$$\delta = \frac{T}{n} = \frac{20}{10,000} = 0.002 \text{ hour}.$$

Why do we consider small interval? The key idea is that as the interval becomes very small, then it is extremely unlikely that there will be more than 1 arrivals during this small amount of time. This statement becomes more accurate as we increase the value of n which decreases the length of each interval ever further. What we are doing here is an approximation of a continuous-time process by a discrete-time process.[4,5]

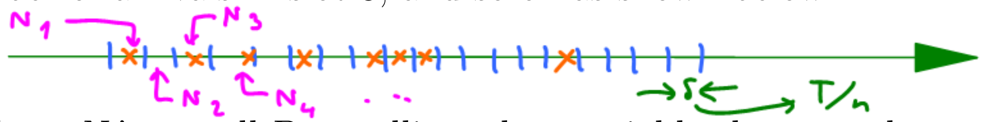
To summarize, we will consider the discrete-time approximation of the (continuous-time) Poisson process. In such approximation, the time axis is divided into many small time intervals (which we call "slots").

When the interval is small enough, we can assume that at most 1 arrival occurs.

[4]You also do this when you plot a graph of any function $f(x)$. You divide the $x-$axis by many (equally spaced) values of x and then evaluate the values of the function at these values of x. You need to make sure that the values of x used are "dense" enough such that no surprising change in the function f is overlooked.

[5]If we wnat to be rigorous, we would have to bound the error from such approximation and show that the error disappear as $n \to \infty$. We will not do that here.

1.7. Let's look at the small slots more closely. Here, we let N_1 be the number of arrivals in slot 1, N_2 be the number of arrivals in slot 2, N_3 be the number of arrivals in slot 3, and so on as shown below.



Then, these N_i's are all Bernoulli random variables because they can only take the values 0 or 1. In which case, for their pmfs, we only need to specify one value $P[N_i = 1]$. Of course, knowing this, we can calculate $P[N_i = 0]$ by $P[N_i = 0] = 1 - P[N_i = 1]$.

Recall that the average $\mathbb{E}X$ of any Bernoulli random variable X is simply $P[X = 1]$.[6] So, if we know $\mathbb{E}X$ for Bernoulli random variable, then we know right away that $P[X = 1] = \mathbb{E}X$ and $P[X = 0] = 1 - \mathbb{E}X$.

Now, it's time to use what we learned about Poisson process. The slots that we consider before are of length T/n. So, the random variables $N_1, N_2, N_3, \ldots$ share the same expected value

$$\mathbb{E}N_1 = \mathbb{E}N_2 = \mathbb{E}N_3 = \cdots = \lambda\delta.$$

For example, with $\lambda = 5$, $T = 20$, and $N = 10,000$, the expected number of arrivals in a slot is

$$\lambda\delta = \lambda\frac{T}{n} = 0.01 \text{ arrivals.}$$

Because these N_i's are all Bernoulli random variables and because they share the same expected value, we can conclude that they are identically distributed; that is their pmf's are all the same. Furthermore, because the slots do not overlap, we also know that the N_i's are independent. Therefore,

the N_i's are i.i.d. Bernoulli random variables whose pmf's are given by

$$p_1 = P[N_i = 1] = \lambda\delta \quad \text{and} \quad p_0 = P[N_i = 0] = 1 - \lambda\delta,$$

where δ is the length of each slot.

1.8. At this point, you can use `MATLAB` to generate a Poisson process with arrival rate λ using discrete-time approximation. Here are the steps:

[6] For Bernoulli random variable X, the average is

$$\mathbb{E}X = 0 \times P[X = 0] + 1 \times P[X = 1] = P[X = 1].$$

For conciseness, we usually let $p_0 = P[X = 0]$ and $p_1 = P[X = 1]$. Hence, $\mathbb{E}X = p_1$.

(a) First, we fix the length T of the whole simulation. (For example, $T = 20$ hours.)

(b) Then, we divide T into n slots. (For example, $n = 10,000$.)

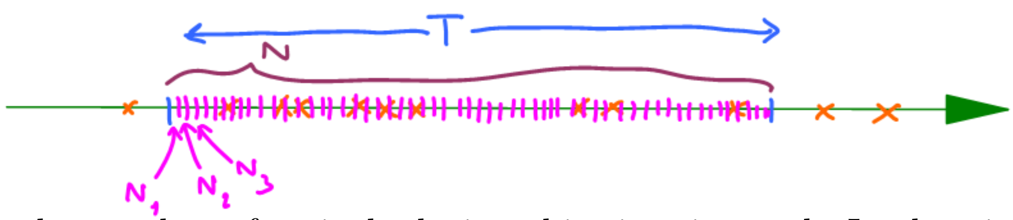
(c) For each slot, only two cases can happen: 1 arrival or no arrival. So, we generate Bernoulli random variable for each slot with $p_1 = \lambda \times T/n$. (For example, if $\lambda = 5$ arrival/hr, then $p_1 = 0.01$.)

To do this for n slots, we can use the command `rand(1,n) < p1` or `binornd(1,p1,1,n)`.

1.9. Note that what we have just generated is exactly ***Bernoulli trials*** whose success probability for each trial is $p_1 = \lambda\delta$. In other words, a Poisson process can be approximated by Bernoulli trials with success probability $p_1 = \lambda\delta$.

1.2 Properties of Poisson Processes

1.10. What we want to do next is to revisit the description of the number of arrivals in a time interval. Now, we will NOT assume that length of the time interval is short. In particular, let's reconsider an interval of length T below.



Let N be the number of arrivals during this time interval. In the picture above, $N = 4$.

Again, we will start with a discrete-time approximation; we divide T into n small slots of length $\delta = \frac{T}{n}$. In the previous subsection, we know that the number of arrivals in these intervals, denoted by $N_1, N_2, \ldots, N_n$ can be well-approximated by i.i.d. Bernoulli with probability of having exactly one arrival $= \lambda\delta$. (Of course, we need δ to be small for the approximation to be precise.) The total number of arrivals during the original interval of length T can be found by summing the values of the N_i's:

$$N \approx N_1 + N_2 + \cdots + N_n. \tag{2}$$

You may recall, from introductory probability class, that

(a) summation of n Bernoulli random variables with success probability p gives a binomial(n, p) random variable[7]

and that

(b) the binomial(n, p) random variable whose n is large and p is small can be well approximated by a Poisson random variable with parameter $\alpha = np$ [8]

Therefore, the pmf of the random variable N in (2) can be approximated by a Poisson pmf whose parameter is

$$\alpha = np_1 = n\lambda\frac{T}{n} = \lambda T.$$

This approximation gets more precise when n is large (δ is small). In fact, in the limit as $n \to \infty$ (and hence $\delta \to 0$), the random variable N is $\mathcal{P}(\lambda T)$. Recall that the expected value of $\mathcal{P}(\alpha)$ is α. Therefore, λT is the expected value of N. This agrees with what we have discussed before in (1).

In conclusion,

> the number N of arrivals in an interval of length T is a Poisson random variable with mean (parameter) λT

1.11. Now, to sum up what we have learned so far, the following is one of the two main properties of a Poisson process

> The number of arrivals $N_1, N_2, N_3, \ldots$ during non-overlapping time intervals are independent Poisson random variables with mean $\lambda \times$ the length of the corresponding interval.

1.12. Another main property of the Poisson process, which we will state without proof, is that

---

[7] X is a ***binomial*** random variable with size $n \in \mathbb{N}$ and parameter $p \in (0, 1)$ if

$$p_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x \in \{0, 1, 2, \ldots, n\} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

We write $X \sim \mathcal{B}(n, p)$ or $X \sim \text{binomial}(p)$. Observe that $\mathcal{B}(1, p)$ is Bernoulli with parameter p. Note also that $\mathbb{E}X = np$.

[8] X is a ***Poisson*** random variable with ***parameter*** $\alpha > 0$ if

$$p_X(k) = \begin{cases} e^{-\alpha}\frac{\alpha^k}{k!}, & k \in \{0, 1, 2, \ldots\} \\ 0, & \text{otherwise} \end{cases}$$

We write $X \sim \mathcal{P}(\alpha)$ or Poisson(α). Note also that $\mathbb{E}X = \alpha$.

> The lengths of time between adjacent arrivals $W_1, W_2, W_3, \ldots$ are i.i.d. exponential[9] random variables with mean $1/\lambda$.

This property can be derived by looking at the discrete-time approximation of the Poisson process. In the discrete-time version, the time until the next arrival is geometric. In the limit, the geometric random variable becomes exponential random variable. Both main properties of Poisson process are shown in Figure 2. The small slot analysis (discrete-time approximation), which can be used to prove the two main properties, is shown in Figure 3.

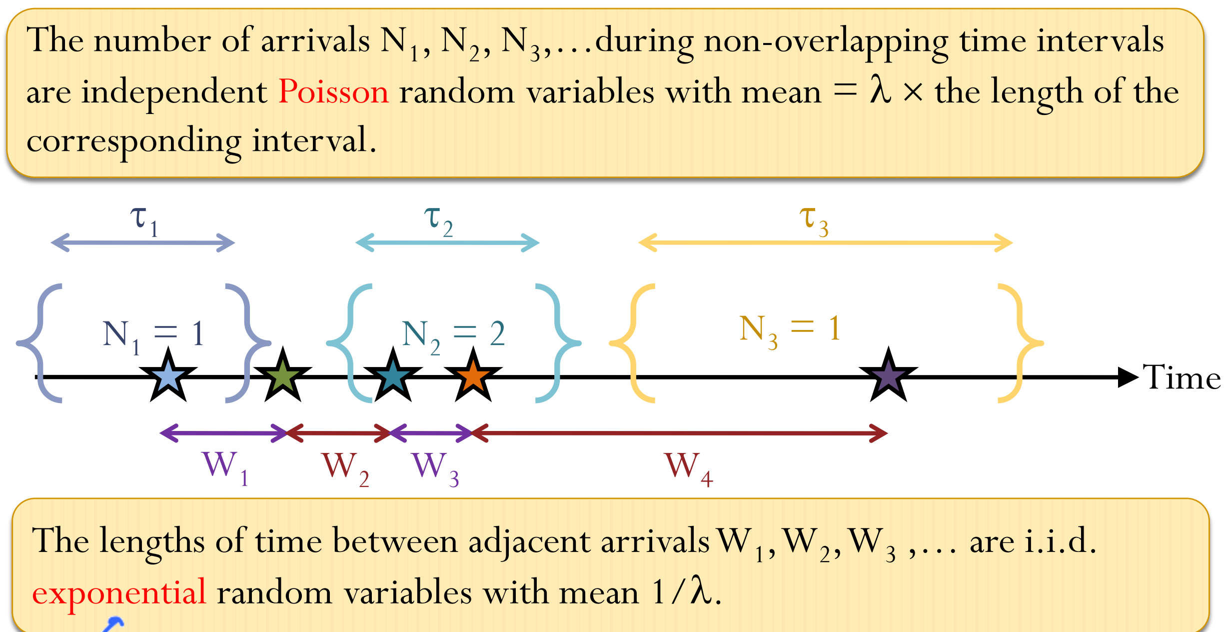


Figure 2: Two main properties of a Poisson process

2 Derivation of the Erlang B Formula

The Erlang B formula along with the definitions of the parameters used in it is shown in Figure 4.

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[9]The exponential distribution is denoted by $\mathcal{E}(\lambda)$. An exponential random variable X is characterized by its probability density function

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

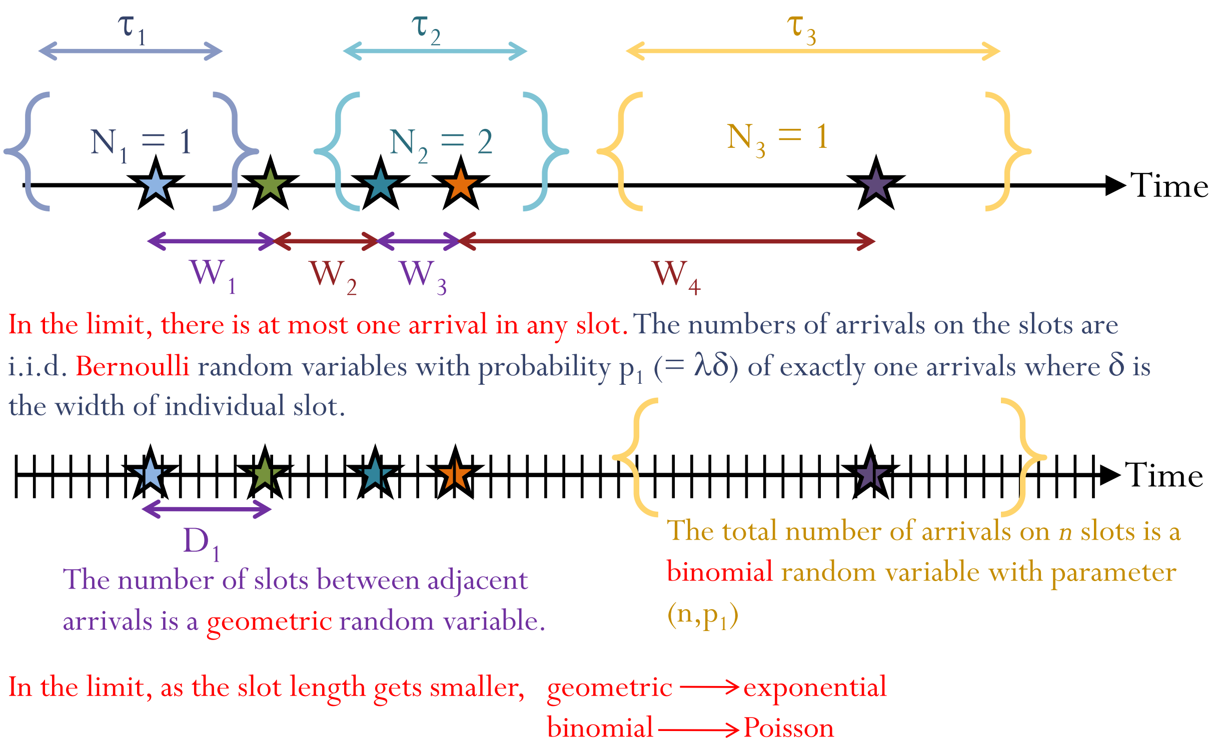


Figure 3: Small slot analysis (discrete-time approximation) of a Poisson process

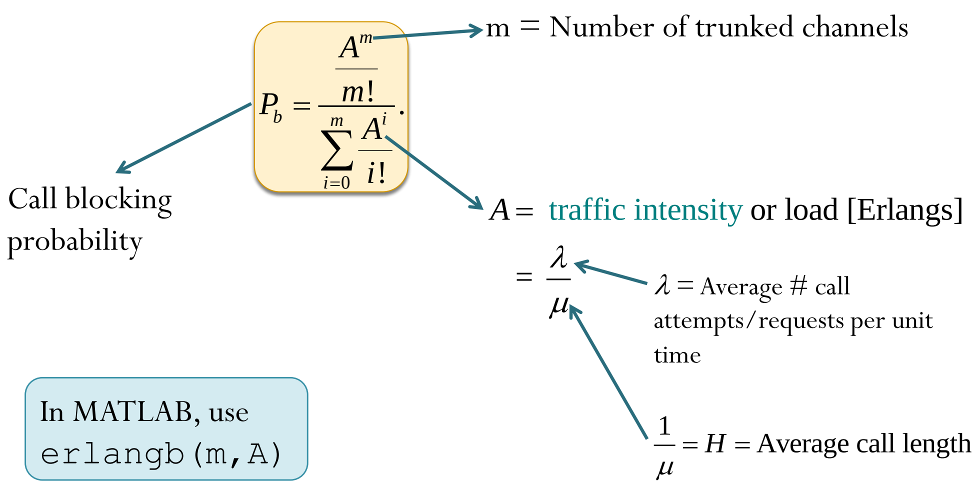


Figure 4: Erlang B Formula

call length or duration $\sim \mathcal{E}(\mu)$

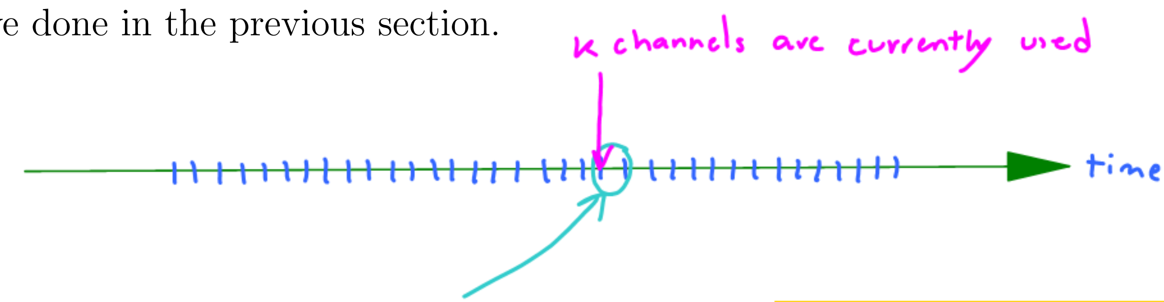
ECS455 2011/2, Chapter 3 Part 2, Dr.Prapun

In the previous section, we discussed Poisson process. In this section, we combine what we know about Poisson process with the assumption on the call length/duration. To do this, we will use (again) the small-slot (discrete-time) approximation.

2.1. For this section, we will assume that there are m channels available in the trunking pool. Therefore, the probability P_b that a call requested by a user will be blocked is given by the probability that none of the m channels are free when a call is requested.

We will consider the long-term behavior of this system, i.e. the system is assumed to have been operating for a long time. In which case, at the instant that somebody is trying to make a call, we don't know how many of the channels are currently free.

2.2. Let's first divide the time into small slots (of the same length δ) as we have done in the previous section.



Then, consider any particular slot. Suppose that at the beginning of this time slot, there are K channels that are currently used.[10] We want to find out how this number K changes as we move forward one slot time. This

[10]The value of K can be any integer from 0 to m.

random variable K will be call the ***state*** of the system[11]. The system moves from one state to another one as we advance one time slot.

Example 2.3. Suppose there are 5 persons using the channels at the beginning of a slot. Then, $K = 5$.

(a) Suppose that, by the end of this slot, none of these 5 persons finish their calls.

(b) Suppose also that there is one new person who wants to make a call at some moment of time during this slot.

Then, at the end of this time slot, the number of channels that are used becomes

$$5 - 0 + 1 = 6.$$

So, the state K of the system changes from 5 to 6 when we reach the end of the slot, which can now be regarded as the beginning of the next slot.

2.4. Our current intermediate goal is to study how the state K changes from one slot to the next slot. Note that it might be helpful to label the state K as K_1 (or $K[1]$) for the first slot, K_2 (or $K[2]$) for the second slot, and so on.

As shown in Example 2.3, to determine how the K_i's progress from K_1 to K_2 to K_3 and so on, we need to know two pieces of information:

Q1 How many calls (that are being made at the beginning of the slot under consideration) ***end*** during the slot that we are considering?

Q2 How many ***new*** call requests are made during the slot under consideration?

Note that Q1 depends on the characteristics of the call duration and Q2 depends on the characteristics of the call request/arrival process. After we know the answers to these two question, then we can find K_i via

$$K_{i+1} = K_i - \underbrace{(\text{\# old call ends})}_{\text{Q1}} + \underbrace{(\text{\# new call requests})}_{\text{Q2}}$$

[11]This is the same "state" concept that you have studied in digital circuits class.

2.5. Q2 is easy to answer.
A2: If the interval are small enough (δ is small), then there can be at most one new arrival (new call request) which occurs with probability

$$p_1 = \lambda\delta.$$

2.6. For Q1, we need to consider the ***call duration model***. The M/M/m/m assumption states that the call duration[12] is exponentially distributed. Let's consider the call duration D of a particular call.

Recall that the probability density function (pdf) of an exponential random variable X with parameter μ is given by

$$f_X(x) = \begin{cases} \mu e^{-\mu x}, & x > 0, \\ 0, & x \le 0. \end{cases}$$

and the average (or expected value) is given by

$$\mathbb{E}[X] = \int_0^\infty x f_X(x)dx = \frac{1}{\mu}.$$

You may remember that in the Erlang B formula, we assume that the average call duration is $\mathbb{E}[D] = H = \frac{1}{\mu}$.

An important property of an exponential random variable X is its memoryless property[13]:

$$P[X > x + \delta | X > x] = P[X > \delta].$$

For example,

$$P[X > 7 | X > 5] = P[X > 2].$$

[12] In queueing theory, this is sometimes called the ***service time***.

[13] To see this, first recall the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Therefore,

$$P[X > x + \delta | X > x] = \frac{P[X > x + \delta \text{ and } X > x]}{P[X > x]} = \frac{P[X > x + \delta]}{P[X > x]}.$$

Now,

$$P[X > x] = \int_x^\infty \mu e^{-\mu x} dx = e^{-\mu x}.$$

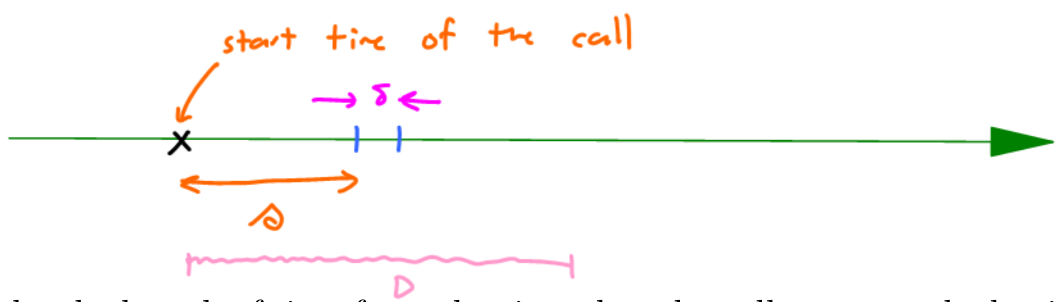
Hence,

$$P[X > x + \delta | X > x] = \frac{e^{-\mu(x+\delta)}}{e^{-\mu x}} = e^{-\mu\delta} = P[X > \delta].$$

What does this memoryless property mean? Suppose you have a lightbulb and you have used it for 5 years already. Let it's lifetime be X. Then, of course, X is a random variable. You know that $X > 5$ because it is still working now. The conditional probability $P[X > 7|X > 5]$ is the probability that it will still work after two more years of use (given the fact that currently it has been working for five years). Now, if X is an exponential random variable, then the memoryless property says that $P[X > 7|X > 5] = P[X > 2]$. In other words, the probability that you can use it for at least two more years is exactly the same as the probability that you can use a new lightbulb for more than two years. So, your old lightbulb essentially forgets that it has been used for 5 years, It always act as a ne lightbulb (statistically). This is we we mean by the memoryless property of an exponential random variable.

2.7. Note that we still haven't answered Q1. We will now return to our small slot approximation. Again, consider one particular slot. At the beginning of our slot, there are $K = k$ ongoing calls. The probability that a particular call, which is still ongoing at the beginning of this slot, will be unfinished by the end of this slot is $e^{-\mu\delta}$.

To see this, consider a particular call. Suppose the duration of this call is D. By assumption, we know that D is exponential with parameter μ.



Let s be the length of time from the time that the call starts to the beginning of our slot. Note that the call is still ongoing. Therefore $D > s$. Now, to say that this call will be unfinished by the end of our slot is equivalent to requiring that $D > s + \delta$. By the memoryless property, we have

$$P[D > s + \delta\,|D > s] = P[D > s] = e^{-\mu\delta}. \quad \approx 1 - \mu\delta$$

Recall that we have $K = k$ ongoing calls at the beginning of our slot. So, by the end of our slot, the probability that none of them finishes is

$$\left(e^{-\mu\delta}\right)^k = e^{-k\mu\delta}. \quad \approx 1 - k\mu\delta$$

$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

The probability that exactly one of them finishes is

$$k \times \left(1 - e^{-\mu\delta}\right) \times \left(e^{-\mu\delta}\right)^{k-1} \approx k \times (\mu\delta) \times \left(1-(k-1)\mu\delta\right) \approx k\mu\delta$$

($e^{-\mu\delta} \approx 1-\mu\delta$)

- k: pick one of the k calls to end in our slot
- $\left(1 - e^{-\mu\delta}\right)$: one call ends
- $\left(e^{-\mu\delta}\right)^{k-1}$: $k-1$ calls are still ongoing by the end of our slot.

Now, note that $e^x \approx 1 + x$ for small x. Therefore, A1:

(a) the probability that none of the $K = k$ calls ends during our slot is

$$\approx 1 - k\mu\delta$$

(b) the probability that exactly one of them ends during our slot is

$$\approx k\mu\delta$$

Magically, these two probabilities sum to one. So, we don't have to consider other events/cases.

2.8. Summary:

call generation/request/initiation process
$P[0 \text{ new call}] \approx 1-\lambda\delta$
$P[1 \text{ new call}] \approx \lambda\delta$

call duration process
$P[0 \text{ old call ends}] \approx 1-k\mu\delta$
$P[1 \text{ old call ends}] \approx k\mu\delta$

So, after one (small) slot, there can be four events:

(i) 0 new call & 0 old-call ends → K stays the same. $(1-\lambda\delta)(1-k\mu\delta)$
(ii) 0 " & 1 " " → K decreases by 1 $(1-\lambda\delta)\,k\mu\delta$
(iii) 1 new call & 0 " " → K increases by 1 $\lambda\delta\,(1-k\mu\delta)$
(iv) 1 " " & 1 " " → K stays the same. $\lambda\delta\;k\mu\delta$

The corresponding probability for each case is

(i) $(1-\lambda\delta)(1-k\mu\delta) \approx 1-\lambda\delta-k\mu\delta$

(ii) $(1-\lambda\delta)k\mu\delta \approx k\mu\delta$

(iii) $\lambda\delta(1-k\mu\delta) \approx \lambda\delta$

(iv) $\lambda\delta \, k\mu\delta \approx 0$

Therefore, if we have $K = k$ at the beginning of our time slot, the at the end of our slot, K may

(a) remain unchanged with probability $1 - \lambda\delta - k\mu\delta$, or

(b) decrease by 1 with probability $k\mu\delta$, or

(c) increase by 1 with probability $\lambda\delta$.

This can be summarized into the following diagram:

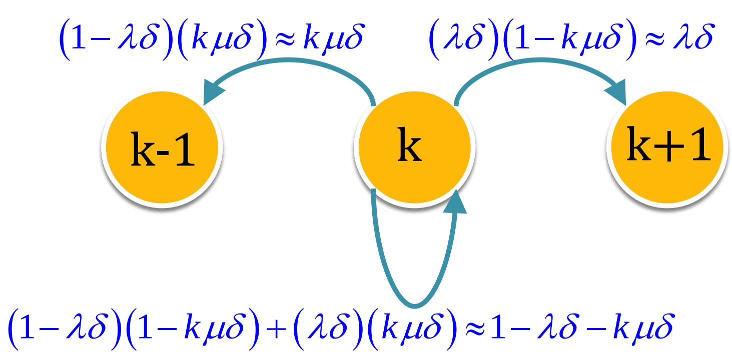
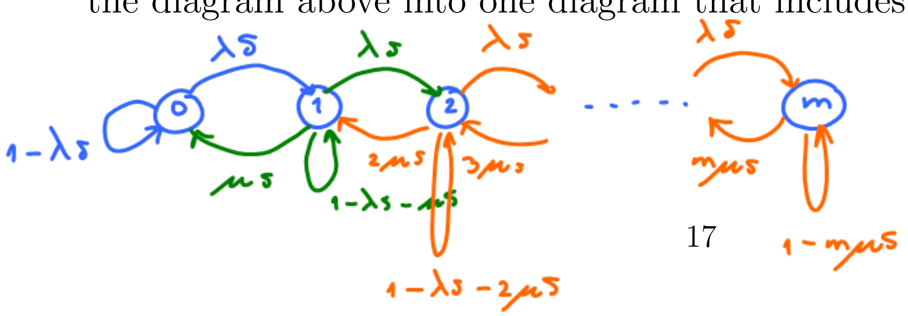


Figure 5: State transition diagram for state k.

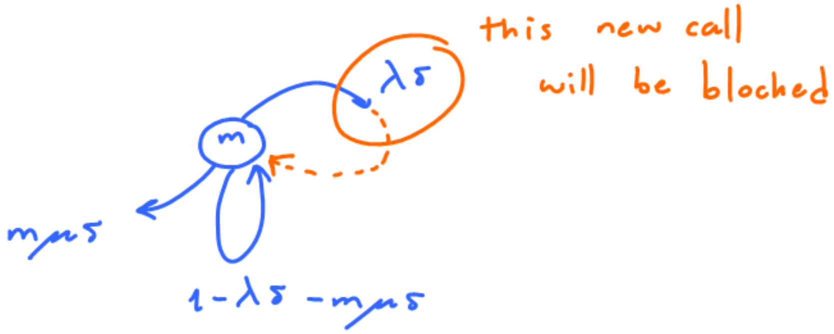
Note that the labels on the arrows indicate transition probabilities which are conditional probabilities of going from some value of K to another value.)

2.9. Given m, the possible values of K are $0, 1, 2, \ldots, m$. We can combine the diagram above into one diagram that includes all possible values of K:

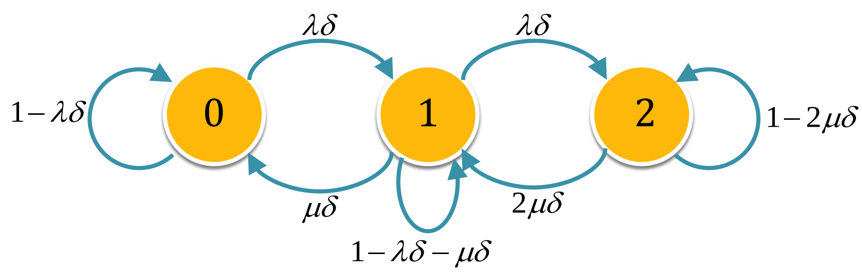


This diagram is called the ***Markov chain state diagram*** for Erlang B.

Note that the arrow $\lambda\delta$ which should go out of state m will return to state m itself because it means blocked calls which do not increase the value of K.



Example 2.10. $m = 2$:



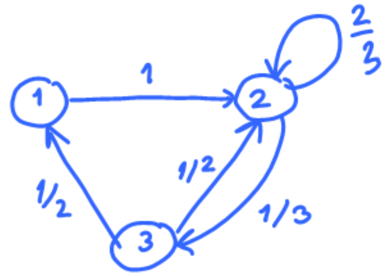
3 Markov Chains

3.1. Markov chains are simple mathematical models for random phenomena evolving in time. Their simple structure makes it possible to say a great deal about their behavior. At the same time, the class of Markov chains is rich enough to serve in many applications. This makes Markv chains the most important examples of random processes. In deed, the whole of the mathematical study of random processes can be regarded as a generalization in one way or another of the theory of Markov chains. [2]

3.2. The characteristic property of Markov chain is that it retains no memory of where it has been in the past. This means that only the current state of the process can influence where it goes next.

3.3. Markov chains are often best described by diagrams.

Example 3.4.

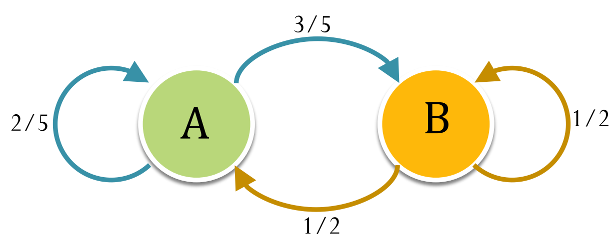


You move from state 1 to state 2 with probability 1. From state 3 you move either to 1 or to 2 with equal probability 1/2, and from 2 you jump to 3 with probability 1/3, otherwise stay at 2.

3.5. The Markov chains that we have seen in the Example 3.4 and in the previous section are all ***discrete-time*** Markov chains. The Poisson process that we have seen earlier is an example of a continuous-time Markov chain. However, with our small-slot approximation (discrete-time approximation), we may analyze the Poisson process as a discrete-time Markov chain.

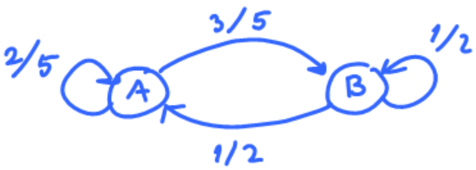
3.6. We will now introduce the concept of stationary distribution, steady-state distribution, equilibrium distribution, and limiting distribution. For the purpose of this class, we will not distinguish these terms. We shall see in the next example that for the Markov chains that we are considering, in the long run, it will reach steady state distribution.

Example 3.7. Consider the Markov chain characterized by the state transition diagram given below:



Let's try a thought experiment – imagine that you start with 100,000 trials of these Markov chain, all of which start in state B. So, during slot 1, all trials will be in state B. For slot 2, about 50% of these will move to state A; but the other 50% of the trials will stay at B.

By the time that you reach slot 6, you can observe that out of the 100,000 trials, about 45.5% will be in state A and about 55.5% will be in state B.



100,000 trials

slot 1 2 3 4 5 6 7 8 9 10

B A A B A A B B A B

B

B

B

B

B

⋮

B

10^5 trials

	A	B
slot 1	0	100,000
slot 2	50,000	50,000
slot 3	20,000 + 25,000 = 45,000	30,000 + 25,000 = 55,000
slot 4	18,000 + 27500 = 45,500	27,000 + 27500 = 54,500
slot 5	45,450	54,550
slot 6	45455	54545
slot 7	45454.5	54545.5
⋮		
slot 1,000	n_A	n_B
1,001	n_A	n_B

(1/2, 1/2; 40%, 60%)

$n_B \frac{1}{2}$ $\quad$ $n_A \frac{3}{5}$

$$\left.\begin{aligned} n_A + n_B &= n \\ \frac{1}{2}\frac{n_B}{n} &= \frac{3}{5}\frac{n_A}{n} \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{1}{2} p_B &= \frac{3}{5} p_A \\ p_A + p_B &= 1 \end{aligned}\right\}$$

$$p_B = \frac{6}{5} p_A$$

$$p_A + \frac{6}{5} p_A = 1 \Rightarrow p_A = \frac{5}{11}$$

These numbers stay roughly the same as you proceed to slot 7, 8, 9, and so on. Note also that it does not matter how you start your 100,000 trials. You may start with 10,000 in state A and 90,000 in state B. Eventually, the same numbers, 45.5% and 54.5%, will emerge.

In conclusion,

(a) If you look at the long run behavior of the Markov chain at a particular slot, then the probability that you will see it in state A is 0.455 and the probability that you will see it in state B is 0.545.

(b) In addition, one can also show that if you look at behavior of a Markov chain for a long time, then the proportion of time that it stays in state A is 45.5% and the proportion of time that it stays in state B is 54.5%.

The distribution $(0.455, 0.545)$ is what we called stationary distribution, steady-state distribution, equilibrium distribution, or limiting distribution above.

3.8. In [3], the steady-state distribution for the M/M/m/m assumption is derived via the use of the ***global balance equation*** instead of finding the limit of the distribution as we have done in Example 3.7. The basic idea that leads to the global balance equation is simple. When we look back at the numbers that we got in Example 3.7, if we assume that they will reach some steady-state values, then at the steady-state, we must have roughly the same number of transitions from state A to B and transitions from state B to state A.

Therefore,

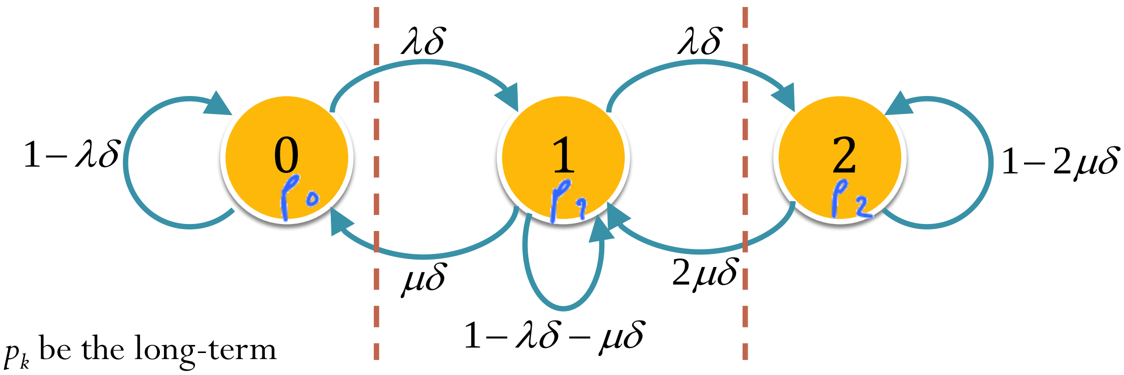
$$n_A + n_B = n, \quad \frac{1}{2}\frac{n_B}{n} = \frac{3}{5}\frac{n_A}{n}$$

$$\frac{1}{2}p_B = \frac{3}{5}p_A, \quad p_A + p_B = 1$$

$$p_B = \frac{6}{5}p_A$$

$$p_A + \frac{6}{5}p_A = 1 \Rightarrow p_A = \frac{5}{11}$$

Finally, we can now use what we learned to derive the Erlang B formula.



Let p_k be the long-term probability that $K = k$.

Global Balance equations

$$\lambda\delta p_0 = \mu\delta p_1 \qquad \lambda\delta p_1 = 2\mu\delta p_2$$

$$p_1 = \frac{\lambda}{\mu} p_0 = A p_0 \qquad p_2 = \frac{\lambda}{2\mu} p_1 = \frac{1}{2} A p_1 = \frac{1}{2} A^2 p_0$$

$$p_0 + p_1 + p_2 = 1$$

$$p_0 + A p_0 + \frac{1}{2} A^2 p_0 = 1$$

$$p_0 = \frac{1}{1 + A + \frac{A^2}{2}}, \quad p_1 = A p_0, \quad p_2 = \frac{1}{2} A^2 p_0$$

$$p_b = p_m$$

$$p_2 = \frac{\frac{1}{2} A^2}{1 + A + \frac{A^2}{2}}$$

Example 3.9. Let's reconsider Example 2.10 where $m = 2$.

The same process can be used to derive the the Erlang B formula.

3.10. In general, if we have m channels, then

$$p_m = \frac{\frac{A^m}{m!}}{\sum_{k=0}^{m} \frac{A^k}{k!}}.$$

Note that p_m is the (long-run) probability that the system is in state m. When the system is in state m, all channels are used and therefore any new call request will be blocked and lost.

Here, p_m is the same as call blocking probability, which is the long-run proportion of call requests that get blocked.

Problem of Erlang B

Engset

Assume infinite users.

λ_u

4 Engsett Model

In this section, we will consider a more realistic system where the infinite-user assumption in M/M/m/m is relaxed.

4.1. Modified (more realistic) assumptions:

(a) Finite number of users: n users/customers

(b) Each user generates new call request with rate λ_u

- Total call request rate $= n \times \lambda_u = \lambda$

4.2. We need to modify our small slot analysis. Here, for each small slot, each user do the following:

(a) If it is idle (not using the channel) at the beginning of the slot,

(i) it may request/generate a new call in a small slot with probability $\lambda_u \delta$.

i. If there is at least one available channel, then it may start its conversation. (In which case, at the beginning of the next slot, its call is ongoing.)

ii. If there is no channel available, then the call is blocked and it is idle again (at the beginning of the next slot).

(ii) With probability $1 - \lambda_u \delta$, no new call is requested by this user during this slot. (In which case, it is idle at the beginning of the next slot.

(b) If it is making a call at the beginning of the slot,

(i) the call may end with probability $\mu\delta$. (In which case, at the beginning of the next slot, it is idle.)

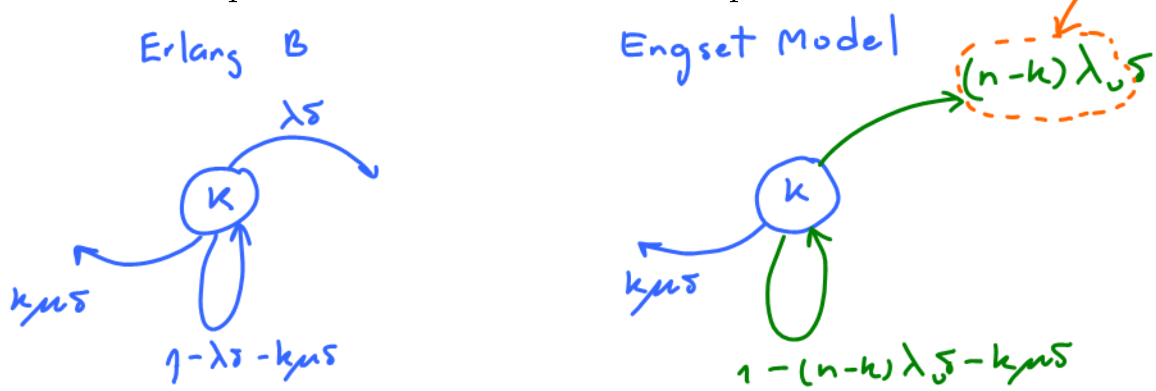
(ii) With probability $1 - \mu\delta$, the call is still ongoing at the end of this slot (which is the same as the beginning of the next slot.)

4.3. Observe that the call generation process for each user is not a Poisson process with rate λ_u. This is because it get interleaved with the call duration for each successful call request. Part of the Poisson assumption that is left is that, in fact, for an idle user, the time until the next call request will be exponential with rate λ_u.

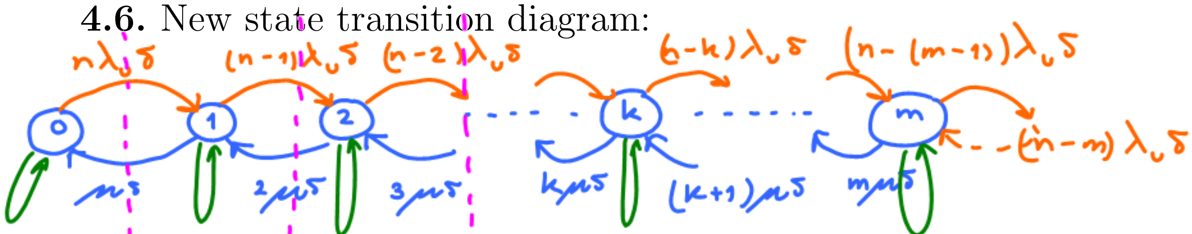
4.4. A user can not generate new call when he/she is already involved in a call. Therefore, if the system is in state $K = k$, there are only $n - k$ users that can generate new calls. Hence, the "total" call request rate for state $K = k$ is $(n-k)\lambda_u$.

- Earlier, when we consider the Erlang B formula, we always have λ as the total new call request rate regardless of how many users are using the channels. This is because we assumed infinite number of users and hence having k users using the channels will not significantly change the total call request rate.

4.5. Comparison of the state transition probabilities:



4.6. New state transition diagram:



$p_0 n\lambda_u\delta = p_1\mu\delta$

$p_1 = nA_u p_0$

$p_1(n-1)\lambda_u\delta = p_2 2\mu\delta$

$p_2 = \frac{n-1}{2}A_u p_1 = \frac{(n-1)n}{2}A_u^2 p_0 = \binom{n}{2}A_u^2 p_0$

$p_2(n-2)\lambda_u\delta = p_3 3\mu\delta$

$p_3 = \frac{n-2}{3}A_u p_2 = \frac{n(n-1)(n-2)}{3\times 2}A_u^3 p_0 = \binom{n}{3}A_u^3 p_0$

$p_k = \binom{n}{k}A_u^k p_0 = \frac{\binom{n}{k}A_u^k}{\sum_{i=0}^{m}\binom{n}{i}A_u^i} = \frac{\binom{n}{k}A_u^k}{z(m,n)}$

$p_0 + p_1 + \cdots + p_m = 1$

$\sum_{k=0}^{m} p_k = 1$

$\sum_{k=0}^{m}\binom{n}{k}A_u^k p_0 = 1$

$p_0 = \frac{1}{\sum_{k=0}^{m}\binom{n}{k}A_u^k}$, where $\sum_{k=0}^{m}\binom{n}{k}A_u^k = z(m,n)$

4.7. Comparison of the steady-state probabilities:

4.8. It is tempting to conclude that the call blocking probability is p_m. However, this is not the case for us here. Recall that p_m is the long-run probability (and the long-run proportion of time) that the system will be in state $K = m$. In this state, any new call request will be blocked. So, p_m gives the blocking probability in terms of the ***time*** (time congestion).

However, if you look back at how we define P_b which is the ***call*** blocking probability, this is the probability that a ***call*** is blocked. So, what we need to find out is, out of all the new calls that are requested, how many will be blocked.

To do this, consider s slots. Here the value of s is very large. Then,...

(a) About $p_k \times s$ slots will be in state k.

(b) Each of these slots will generate new call request with probability $(n - k)\lambda_u\delta$.

(c) So, the number of new calls request from slots that are in state k will be approximately

$$(n-k)\lambda_u\delta \times (p_k \times s).$$

(d) Therefore, total number of new call requests will be approximately

$$\sum_{k=1}^{m}(n-k)\lambda_u\delta \times (p_k \times s).$$

(e) However, the number of the new call requests that get blocked is

$$(n-m)\lambda_u\delta \times (p_m \times s).$$

(f) Hence, the proportion (probability) of calls that are blocked is

$$\frac{(n-m)\lambda_u\delta \times (p_m \times s)}{\sum_{k=0}^{m}(n-k)\lambda_u\delta \times (p_k \times s)} = \frac{(n-m)p_m}{\sum_{k=0}^{m}(n-k)p_k}.$$

Plugging in the values of p_k and p_m which we got earlier, we then get

$$P_b = \frac{(n-m)p_m}{\sum\limits_{k=0}^{m}(n-k)p_k} = \frac{(n-m)\frac{A_u^m\binom{n}{m}}{z(m,n)}}{\sum\limits_{k=0}^{m}(n-k)\frac{A_u^k\binom{n}{k}}{z(m,n)}} = \frac{(n-m)A_u^m\binom{n}{m}}{\sum\limits_{k=0}^{m}(n-k)A_u^k\binom{n}{k}}.$$

4.9. Comparison of the call blocking probability:

Erlang B

$$P_b = \frac{\lambda p_m}{\sum_{k=0}^{m}\lambda p_k} = p_m$$

Engset

$$P_b = \frac{(n-m)p_m}{\sum_{k=0}^{m}(n-k)p_k}$$

4.10. Remarks: $\lambda_u = \frac{\lambda}{n}$

(a) If we keep the total rate λ constant and let $n \to \infty$, then the call blocking probability in the Engsett model will be the same as the call blocking probability in the Erlang model. [HW3]

(b) If $n \leq m$, the call blocking probability in Engsett model will be 0.

(c) M/M/m/m for Erlang B formula (M: Poisson arrival process; M: call duration process; m: # channels)

In HW3, we will see M/M/m/∞ → Erlang C.

References

[1] Andrea Goldsmith. *Wireless Communications*. Cambridge University Press, 2005. 2

[2] J. R. Norris. *Markov Chains*. Cambridge University Press, 1998. 3.1

[3] Theodore S. Rappaport. *Wireless Communications: Principles and Practice*. Prentice Hall PTR, 2 edition, 2002. 3.8

Mobile Communications

ECS 455

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Part II

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ECS455: Chapter 4

Multiple Access

Resource sharing
- different users ← 4.3
- uplink/downlink (duplexing) ← 4.1

Office Hours:
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Friday 9:30-10:30

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Parameter	Fixed WiMAX	Mobile WiMAX	HSPA	1x EV-DO Rev A	Wi-Fi
Standards	IEEE 802.16-2004	IEEE 802.16e-2005	3GPP Release 6	3GPP2	IEEE 802.11a/g/n
Peak down link data rate	9.4Mbps in 3.5MHz with 3:1 DL-to-UL ratio TDD; 6.1Mbps with 1:1	46Mbps[a] with 3:1 DL- to-UL ratio TDD; 32Mbps with 1:1	14.4Mbps using all 15 codes; 7.2Mbps with 10 codes	3.1Mbps; Rev. B will support 4.9Mbps	54 Mbps[b] shared using 802.11a/g; more than 100Mbps peak layer 2 through-put using 802.11n
Peak uplink data rate	3.3Mbps in 3.5MHz using 3:1 DL-to-UL ratio; 6.5Mbps with 1:1	7Mbps in 10MHz using 3:1 DL-to-UL ratio; 4Mbps using 1:1	1.4Mbps initially; 5.8Mbps later	1.8Mbps	
Bandwidth	3.5MHz and 7MHz in 3.5GHz band; 10MHz in 5.8GHz band	3.5MHz, 7MHz, 5MHz, 10MHz, and 8.75MHz initially	5MHz	1.25MHz	20MHz for 802.11a/g; 20/40MHz for 802.11n
Modulation	QPSK, 16 QAM, 64 QAM	QPSK, 16 QAM, 64 QAM	QPSK, 16 QAM	QPSK, 8 PSK, 16 QAM	BPSK, QPSK, 16 QAM, 64 QAM
Multiplexing	TDM	TDM/OFDMA	TDM/CDMA	TDM/ CDMA	CSMA
Duplexing	TDD, FDD	TDD initially	FDD	FDD	TDD
Frequency	3.5GHz and 5.8GHz initially	2.3GHz, 2.5GHz, and 3.5GHz initially	800/900/1,800/ 1,900/ 2,100MHz	800/900/ 1,800/ 1,900MHz	2.4GHz, 5GHz
Coverage (typical)	3–5 miles	< 2 miles	1–3 miles	1–3 miles	< 100 ft indoors; < 1000 ft outdoors
Mobility	Not applicable	Mid	High	High	Low

Of interest for consumer.

← Digital commu.

ECS455: Chapter 4

Multiple Access

4.1 TDD and FDD

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Friday 14:00-16:00

Duplexing

- Allow the subscriber to send "simultaneously" information to the base station while receiving information from the base station.
  - Talk and listen simultaneously.
- Definitions:
  - **Forward channel** or **downlink (DL)** is used for communication from the infrastructure to the users/stations
  - **Reverse channel** or **uplink (UL)** is used for communication from users/stations back to the infrastructure.
- Two techniques
  1. Frequency division duplexing (FDD)
  2. Time division duplexing (TDD)

[Rappaport, 2002, Ch 9]

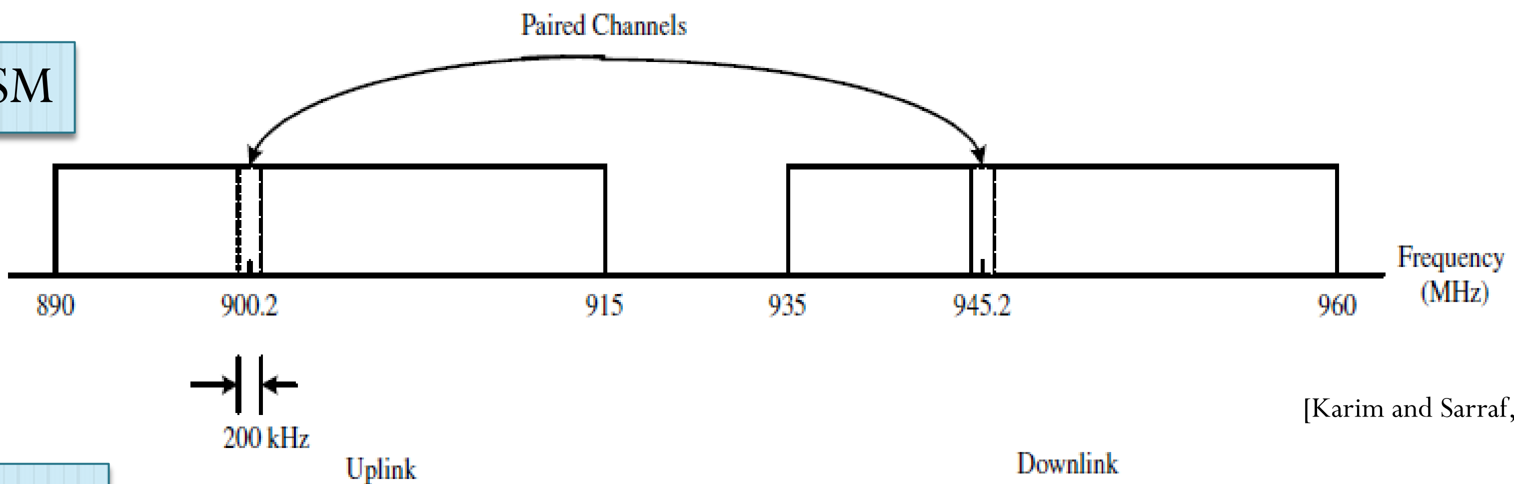
Frequency Division Duplexing (FDD)

- Provide ***two distinct bands*** of frequencies (simplex channels) for every user.
- The **forward band** provides traffic from the base station to the mobile.
- The **reverse band** provides traffic from the mobile to the base station.
- Any *duplex* channel actually consists of two *simplex* channels (a forward and reverse).
- Most commercial cellular systems are based on FDD.

FDD Examples

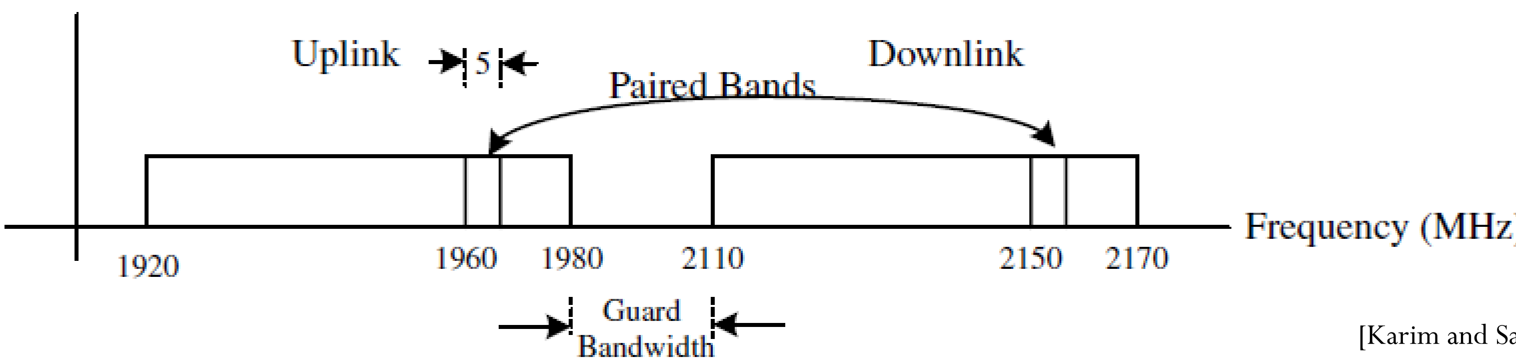
Q: why the two frequencies in a paired channel are so far apart??

GSM



[Karim and Sarraf, 2002, Fig 5-1]

UMTS



[Karim and Sarraf, 2002, Fig 6-1]

Time Division Duplexing (TDD)

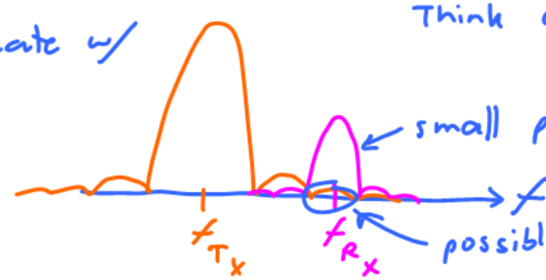
- The UL and DL data are transmitted on the **same carrier frequency** at different times. (Taking turns)
  - Use time instead of frequency to provide both forward and reverse links.
  - Each *duplex* channel has both a **forward time slot** and a **reverse time slot**.
- If the *time separation* between the forward and reverse lime slot is *small*, then the transmission and reception of data *appears* simultaneous to the users at both the subscriber unit and on the base station side.
- Used in Bluetooth and Mobile WiMAX
- Each transceiver operates as either a transmitter or receiver on the same frequency

Problems of FDD

- Each transceiver simultaneously transmits and receives radio signals
  - The signals transmitted and received can vary by more than 100 dB.
  - The signals in each direction need to occupy bands that are ***separated far apart*** (tens of MHz)
- A device called a **duplexer** is required to filter out any interference between the two bands.

Note that Tx, Rx for a user is @ the same location.
(Rx: receive sth from BS; Tx: communicate w/ BS)

Think about power @ the mobile terminal



[Tse and Viswanath, 2005, Ch 4, p 121]

Advantages of FDD (Bad pts for TDD)

- TDD frames need to incorporate guard periods **equal to the max round trip propagation delay** to avoid interference between uplink and downlink under worst-case conditions.
- There is a **time latency** created by TDD due to the fact that communications is not full duplex in the truest sense.
  - This latency creates inherent sensitivities to propagation delays of individual users.

Advantages of TDD

- Duplexer is not required.
- Enable easier *adjustment* of the downlink/uplink ratio to efficiently support *asymmetric* DL/UL traffic.
  - With FDD, DL and UL always have fixed and generally, equal DL and UL *bandwidths*.
- Assure ***channel reciprocity*** for better support of link adaptation, MIMO and other closed loop advanced antenna technologies.
- Ability to implement in ***nonpaired spectrum***
  - FDD requires a pair of channels
  - TDD only requires a single channel for both DL and UL providing greater flexibility for adaptation to varied global spectrum allocations.

ECS455: Chapter 4

Multiple Access

4.2 Introduction to Multiple Access

Parameter	Fixed WiMAX	Mobile WiMAX	HSPA	1x EV-DO Rev A	Wi-Fi
Standards	IEEE 802.16-2004	IEEE 802.16e-2005	3GPP Release 6	3GPP2	IEEE 802.11a/g/n
Peak down link data rate	9.4Mbps in 3.5MHz with 3:1 DL-to-UL ratio TDD; 6.1Mbps with 1:1	46Mbps[a] with 3:1 DL- to-UL ratio TDD; 32Mbps with 1:1	14.4Mbps using all 15 codes; 7.2Mbps with 10 codes	3.1Mbps; Rev. B will support 4.9Mbps	54 Mbps[b] shared using 802.11a/g; more than 100Mbps peak layer 2 through-put using 802.11n
Peak uplink data rate	3.3Mbps in 3.5MHz using 3:1 DL-to-UL ratio; 6.5Mbps with 1:1	7Mbps in 10MHz using 3:1 DL-to-UL ratio; 4Mbps using 1:1	1.4Mbps initially; 5.8Mbps later	1.8Mbps	
Bandwidth	3.5MHz and 7MHz in 3.5GHz band; 10MHz in 5.8GHz band	3.5MHz, 7MHz, 5MHz, 10MHz, and 8.75MHz initially	5MHz	1.25MHz	20MHz for 802.11a/g; 20/40MHz for 802.11n
Modulation	QPSK, 16 QAM, 64 QAM	QPSK, 16 QAM, 64 QAM	QPSK, 16 QAM	QPSK, 8 PSK, 16 QAM	BPSK, QPSK, 16 QAM, 64 QAM
Multiplexing	TDM	TDM/OFDMA	TDM/CDMA	TDM/ CDMA	CSMA
Duplexing	TDD, FDD	TDD initially	FDD	FDD	TDD
Frequency	3.5GHz and 5.8GHz initially	2.3GHz, 2.5GHz, and 3.5GHz initially	800/900/1,800/ 1,900/ 2,100MHz	800/900/ 1,800/ 1,900MHz	2.4GHz, 5GHz
Coverage (typical)	3–5 miles	< 2 miles	1–3 miles	1–3 miles	< 100 ft indoors; < 1000 ft outdoors
Mobility	Not applicable	Mid	High	High	Low

Multiple Access Techniques

- Allow **many** mobile users to **share** simultaneously a finite amount of radio spectrum.
- For high quality communications, this must be done without severe degradation in the performance of the system.
- Important access techniques
  1. Frequency division multiple access (FDMA)
  2. Time division multiple access (TDMA)
  3. Spread spectrum multiple access (SSMA)
     - Frequency Hopped Multiple Access (FHMA)
     - Code division multiple access (CDMA)
  4. Space division multiple access (SDMA)
  5. Random access
     - ALOHA

we have already seen this

freq. reuse

Sectoring

Chapter 4
Multiple Access

4.3 FDMA and TDMA

Multiple Access Techniques

- Allow **many** mobile users to **share** simultaneously a finite amount of radio spectrum.
- For high quality communications, this must be done without severe degradation in the performance of the system.
- Important access techniques
  1. Frequency division multiple access (FDMA)
  2. Time division multiple access (TDMA)
  3. Spread spectrum multiple access (SSMA)
     - Frequency Hopped Multiple Access (FHMA)
     - Code division multiple access (CDMA)
  4. Space division multiple access (SDMA)
  5. Random access
     - ALOHA

Frequency division multiple access (FDMA)

- The *oldest* multiple access scheme for wireless communications.
- Used exclusively for multiple access in 1G down to individual resource units or physical channels.
- Assign individual channels to individual users.
  - Different carrier frequency is assigned to each user so that the resulting spectra do not overlap.
  - During the period of the call, no other user can share the same channel.
- **Band-pass filtering** (or heterodyning) enables separate demodulation of each channel.

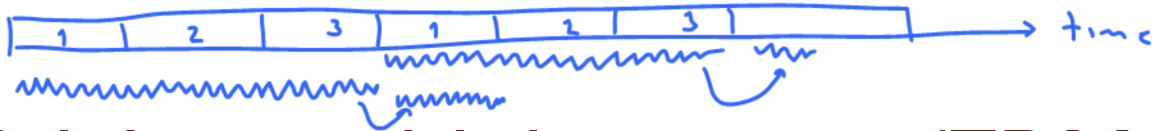
FDMA (2)

- If an FDMA channel is not in use, then it sits idle and can**not** be used by other users to increase or share capacity.
  - It is essentially a wasted resource.

FDMA/FDD

- In FDD systems, the users are assigned a channel as a pair of frequencies.

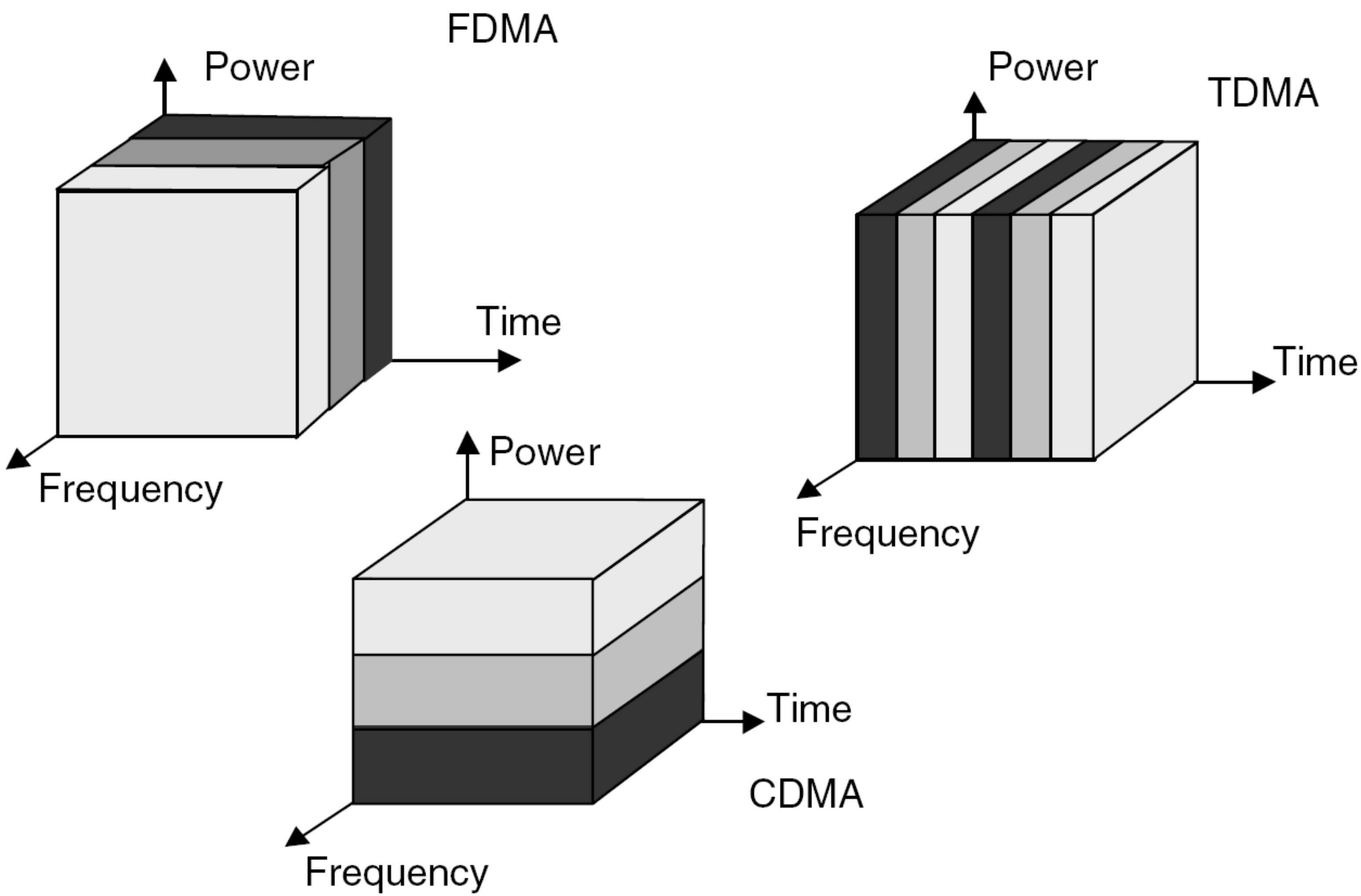
[Rappaport, 2002, Ch 9, p. 449]



Time division multiple access (TDMA)

- Divide the radio spectrum into **time slots**.
- In each slot only one user is allowed to either transmit or receive.
- A channel may be thought of as a particular time slot that reoccurs every frame, where *N* time slots comprise a frame.
- Transmit data in a **buffer-and-burst method**
  - The transmission for any user is non-continuous.
  - Digital data and digital modulation must be used with TDMA.
  - This results in low battery consumption, since the subscriber transmitter can be turned off when not in use (which is most of the time).
- An obvious choice in the 1980s for digital mobile communications.

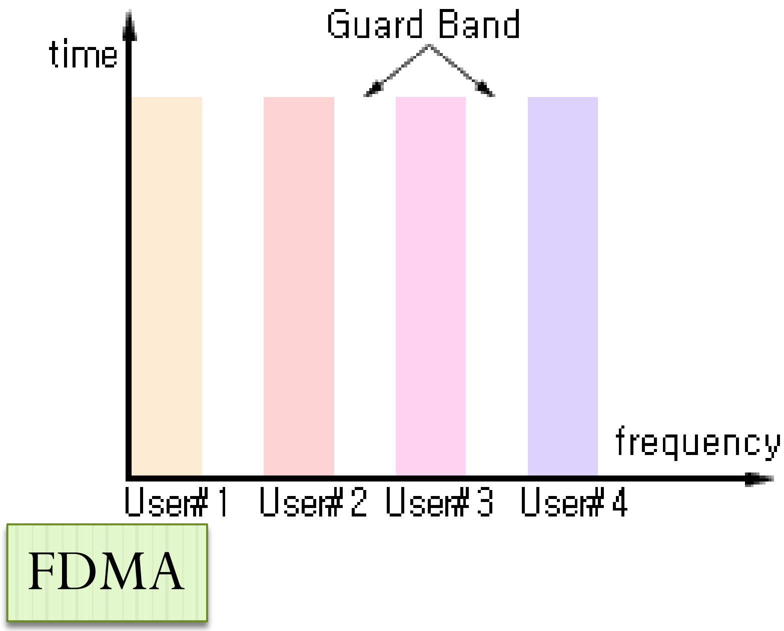
FDMA vs. TDMA

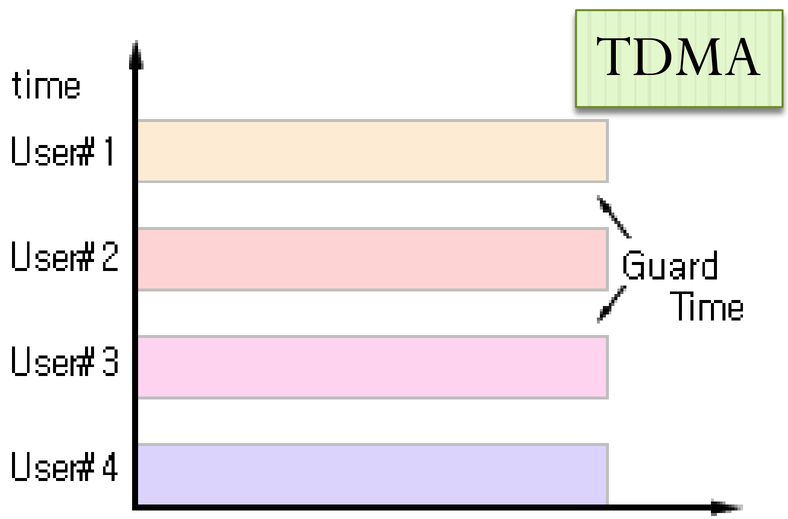

FDMA
Power
Time
Frequency
Power
TDMA
Time
Frequency
Power
Time
CDMA
Frequency

Tradeoffs

- TDMA transmissions are slotted
  - Require the receivers to be **synchronized** for each data burst.
  - **Guard times** are necessary to separate users. This results in larger overheads.
  - FDMA allows completely **uncoordinated transmission** in the time domain
    - No time synchronization among users is required.
- The complexity of FDMA mobile systems is lower when compared to TDMA systems, though this is changing as digital signal processing methods improve for TDMA.
- Since FDMA is a continuous transmission scheme, fewer bits are needed for **overhead** purposes (such as synchronization and framing bits) as compared to TDMA.
- FDMA needs to use costly **bandpass filters**.
  - For TDMA, no filters are required to separate individual physical channels.

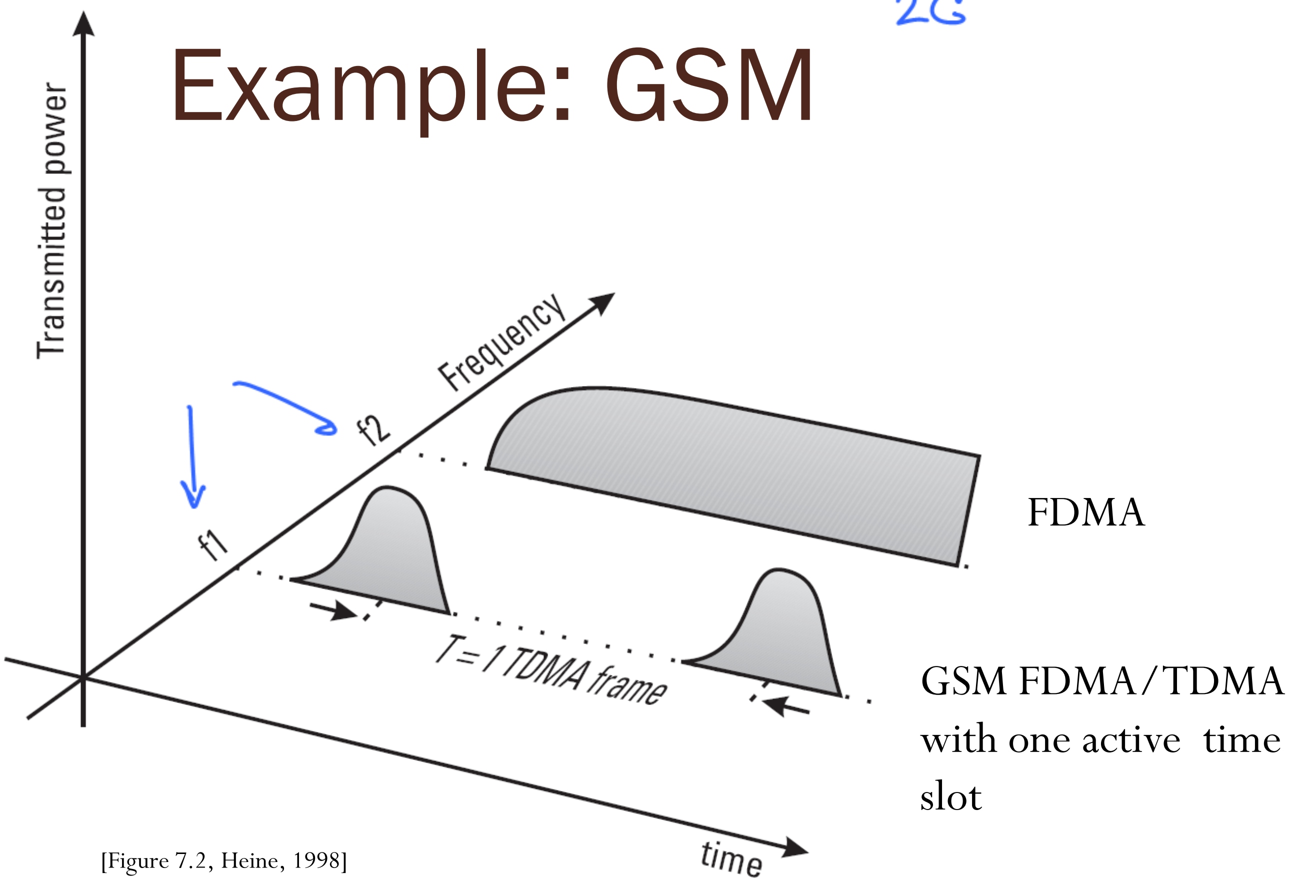
Guard Band vs. Guard Time





Example: GSM

2G



[Figure 7.2, Heine, 1998]

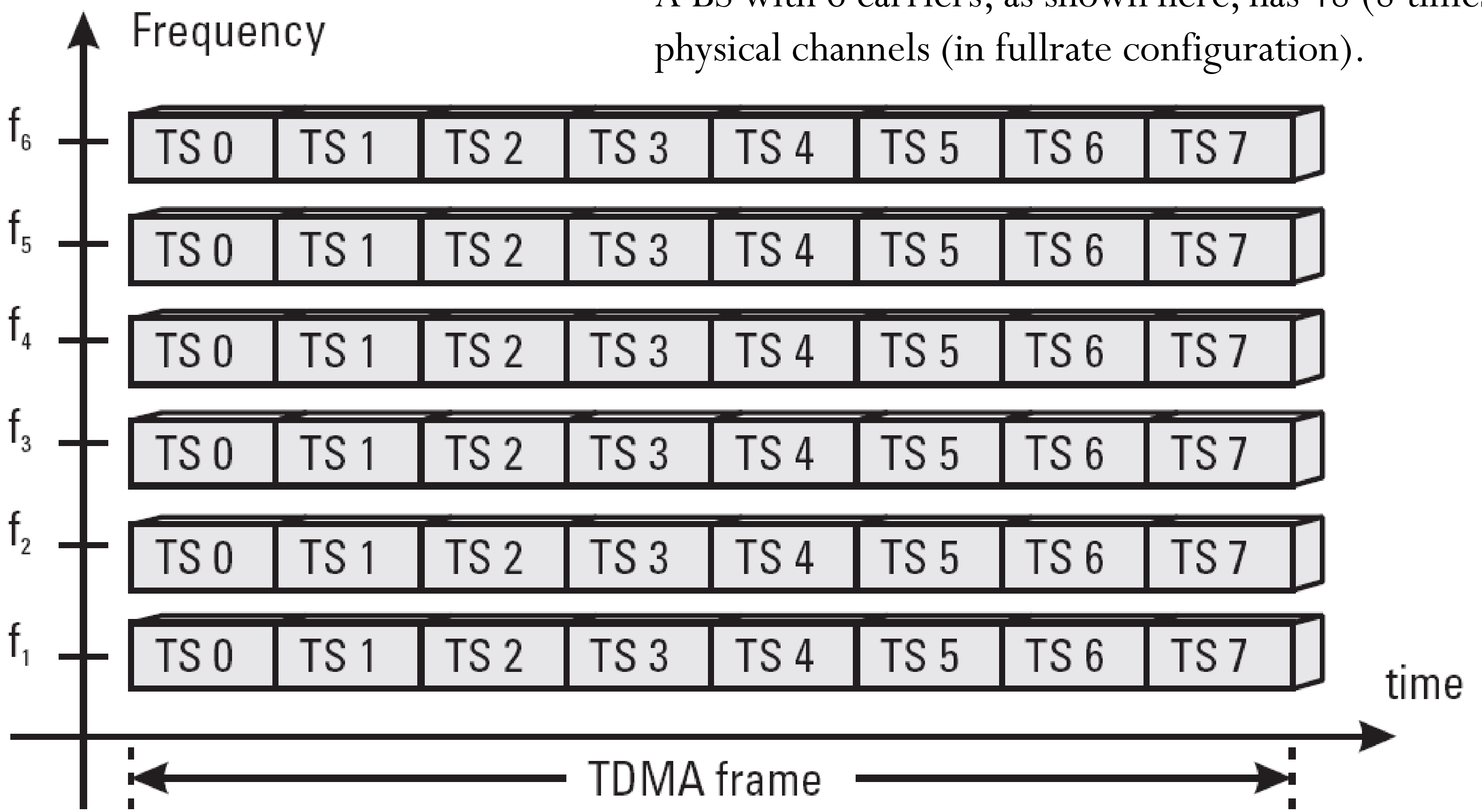
- GSM utilizes a combination of FDMA and TDMA
- Two-dimensional channel structure

- Each narrowband channel has bandwidth 200 kHz.
- Time is divided into slots of length T =577 μs.

The FDMA/TDMA structure of GSM

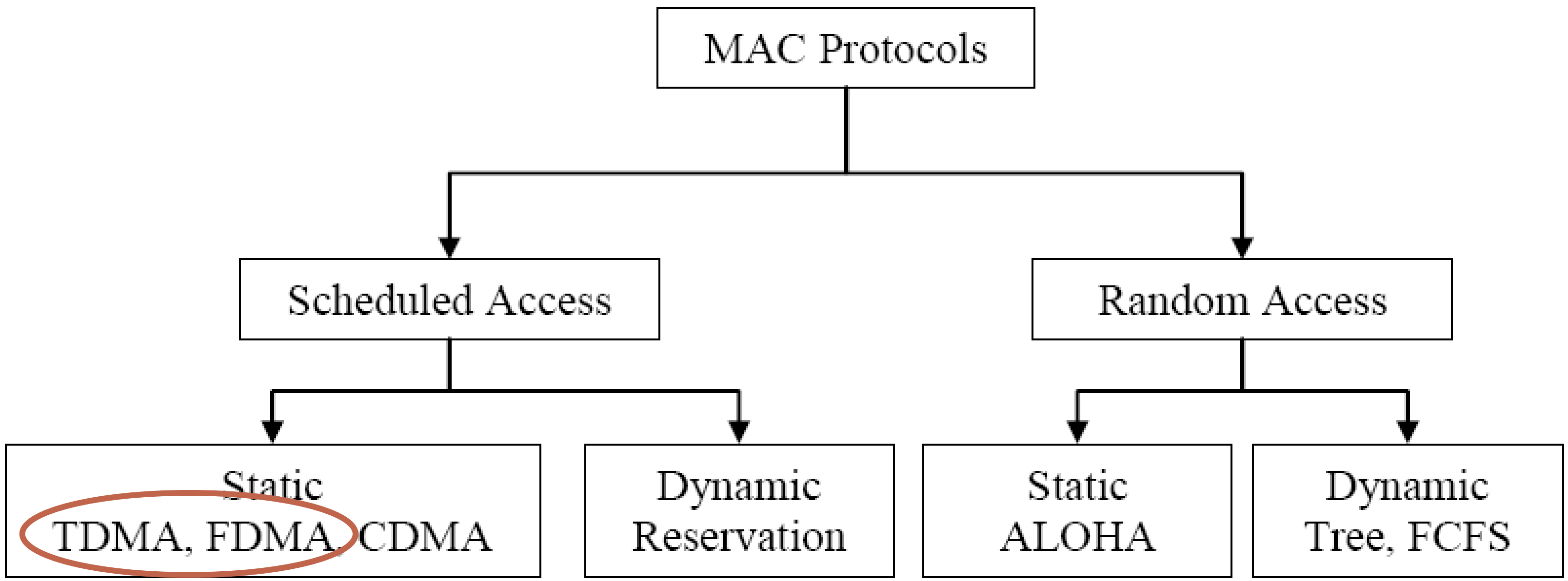
- In full-rate configuration, eight time slots (TSs) are mapped on every frequency.

A BS with 6 carriers, as shown here, has 48 (8 times 6) physical channels (in fullrate configuration).



[Figure 7.1, Heine, 1998]

Classifications of Medium Access Control (MAC)



Cellular System	Multiple Access Technique
Advanced Mobile Phone System (AMPS)	FDMA/FDD
Global System for Mobile (GSM)	TDMA/FDD
US Digital Cellular (USDC)	TDMA/FDD
Pacific Digital Cellular (PDC)	TDMA/FDD
CT2 (Cordless Telephone)	FDMA/TDD
Digital European Cordless Telephone (DECT)	FDMA/TDD
US Narrowband Spread Spectrum (IS-95)	CDMA/FDD
W-CDMA (3GPP)	CDMA/FDD CDMA/TDD
cdma2000 (3GPP2)	CDMA/FDD CDMA/TDD